Department of Physics – Middle East Technical University

PHYS 701 Practice Exam

Name:		Student ID #:	

Please read the rules below carefully:

- This is an open-resource test; however, don't consult anyone. This action will be considered cheating.
- Make sure that you use your own words when you get information from a source! Copying sentences is an act of plagiarism! This includes AI tools as well.
- Keep your answers concise; writing too much doesn't always show that you know much.
- You have 270 minutes to answer 6 questions.
- Email your solutions back to me by 21:30 today. For each 5 minutes you are late, you will lose 10 points! You can send JPEG images of your solutions. Make sure that they are well-lit and legible.
- For integrals, use https://www.wolframalpha.com/

Some fundamental constants and possibly useful equations:

Planck's constant: $\boldsymbol{h = 6.63\ 10^{-34}\ J.s}$ - $\boldsymbol{4.14\ 10^{-15}\ eV.s}$

Reduced Planck's constant: $\boldsymbol{\hbar = 1.05\ 10^{-34}\ J.s\ -\ 6.58\ 10^{-16}\ eV.s}$

Speed of Light in vacuum: $\boldsymbol{c = 3\ 10^{8}\ m/s}$

Electron mass: $\boldsymbol{m = 9.1\ 10^{-31}\ kg\ -\ 0.511\ MeV/c^{2}}$

-$\boldsymbol{c = \lambda f}$ - $\boldsymbol{E = hf}$ - $\boldsymbol{V = IR}$ - $\boldsymbol{E = mc^{2}}$

1) Please answer the following questions related to quantum mechanics.
    a. Write down the Schrödinger's equation.
    b. Explain what wave function is.
    c. Why wave function corresponding to a physical system must be normalizable?
    d. A particle in *an infinite potential well* has the following wave function:

$$\boldsymbol{\Psi(x, 0) = A[\psi_1 + e^{i\phi}\psi_3]}$$

where $\boldsymbol{\phi}$ is some constant. $\boldsymbol{\psi_i}$ corresponds to the i^th stationary state.

i. First, normalize the wave function.
ii. Then, find the probability of finding the system in the 1st and the 3rd states.
iii. Calculate $\boldsymbol{\Psi(x, t)}$.

 Name:

2) Bloch's theorem plays a central role in the study of quantum crystalline solids.
    a. Explain what Bloch theorem is.
    b. Write down the wave function for a Bloch wave and show that the probability density has the same periodicity as the potential energy of the system.
    c. Explain how Bloch's theorem is related to our understanding of the electronic properties of the materials.

3) The band model we studied for the nearly free electron Fermi gas states that unlike the free electron Fermi gas, when all the accessible states are filled at a band, nothing changes when an external electric field is applied. When the band gap is small enough, we name such materials as semiconductors.
    a. What is Fermi Level, Fermi Energy and Fermi Wavelength?
    b. Explain the difference between direct and indirect gap semiconductors. Why direct gap semiconductors are preferred over indirect gap ones for the optoelectronics applications?
    c. Explain the electrical conduction mechanism in an intrinsic semiconductor.
    d. When we talk about doping of a semiconductor what changes in the crystal and what happens to the Fermi Level for different dopants.
    e. Explain how we can create a 2D electron gas using a Si-MOSFET.

4) Hall Effect is a very important tool for measuring the charge density of materials. Let's assume we would like to take Hall Effect measurements from a semiconductor with electron and hole densities of n and p, relaxation times τ_e and τ_h and effective masses m_e and m_h. a. Show that the Hall coefficient is:

$$R_H = \frac{1}{e}\frac{\left(p - n\left(\frac{\mu_e}{\mu_h}\right)^2\right)}{\left(p + n\frac{\mu_e}{\mu_h}\right)^2}$$

This is not a straightforward question! Please make sure that you set up the problem correctly. You need to ignore higher order terms in B such as B^2 and so on. Make sure that you have the expressions for the conductivity in terms of densities, relaxation times and effective masses.

b. What happens to the Hall coefficient in the intrinsic case?

5) Density of states in a free electron Fermi gas depends on the dimensionality of the system.
   a. First, derive the density of states for a 3D box.
   b. Then, a 2D box
   c. and, finally a 1D box.
   d. For each case, how would the electronic states fill up with the increasing temperature? Draw a simple schematic to show how the states will be filled. Your schematic should clearly point the position of the Fermi energy.
   e. Show that for an intrinsic semiconductor Fermi level always lies in between the energy gap. This demonstration requires writing n and p in terms of the integral of density of states with the Fermi function. Integral limits should satisfy the bottom of the conduction band to infinity and top of the valance band to negative infinity. *This one is probably the most algebraically involved question in this midterm. If you don't have time and left other questions, use your time wisely!*

 Name:

6) Explain Landau levels in detail.

	1	2	3	4	5	6	Total
Max Pts	*15*	*15*	*20*	*15*	*20*	*15*	*100*
Earned							