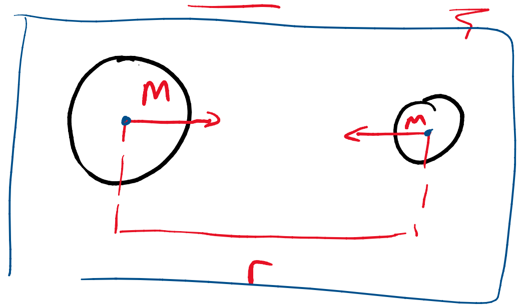
Lecture 24

19 Aralık 2019 Perşembe 08:42

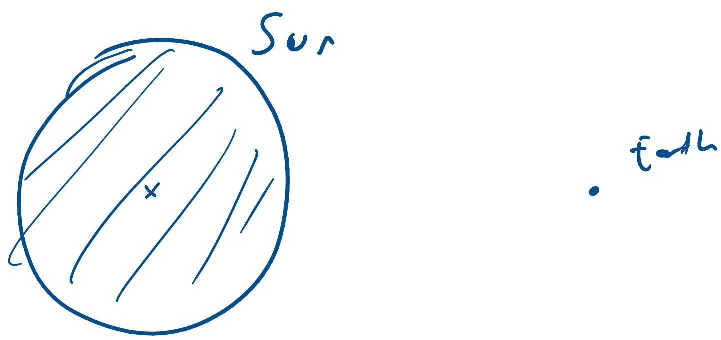
⇒ We Newtonian gravity → $|\vec{F}| = G\frac{Mm}{r^2}$, $U = -G\frac{Mm}{r}$

$\boxed{-\vec{\nabla} U = \vec{F}}$

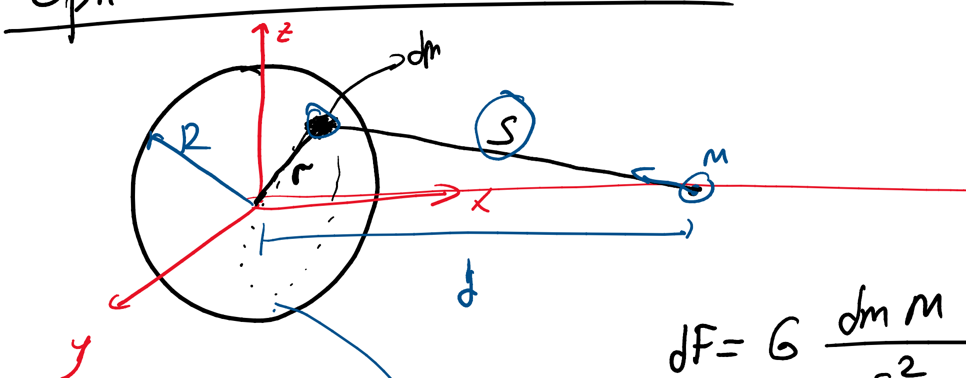


○ Kepler's laws

○ Black holes , Einsteinian gravity

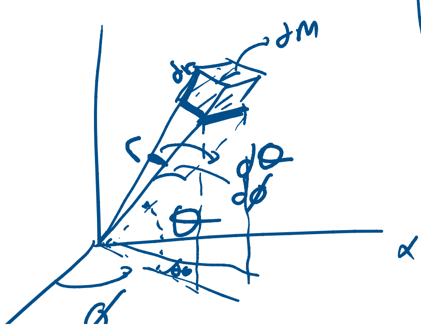


Spherical Mass Distributions



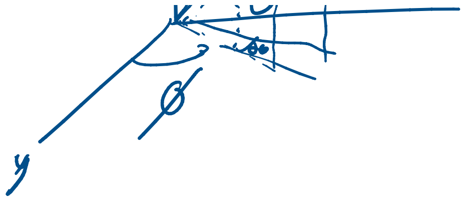
$$dF = G\frac{dm\,M}{s^2}$$

$$dU = -G\frac{dm\,M}{s}$$



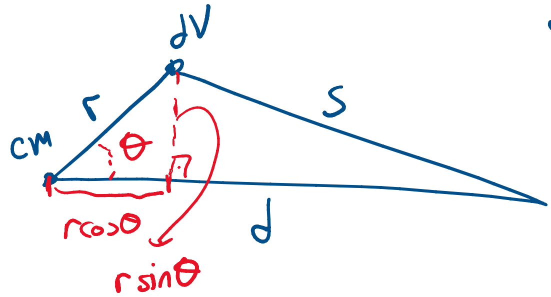
$$\frac{M}{\frac{4}{3}\pi R^3} = \rho$$

$$dm = \rho\, dV = \rho \cdot \underbrace{dr\, r}\, d\theta\, r\sin\theta \cdot d\phi$$



$\propto \quad \frac{4}{3}\pi R$

$$s^2 = r^2\sin^2\theta + (d - r\cos\theta)^2$$



$$s^2 = \underbrace{r^2\sin^2\theta} + d^2 - 2rd\cos\theta + \underbrace{r^2\cos^2\theta}$$

$$s^2 = r^2(\underbrace{\sin^2\theta + \cos^2\theta}_{1}) - 2rd\cos\theta + d^2$$

Law's cosines $\longrightarrow$ $\boxed{s^2 = r^2 - 2rd\cos\theta + d^2}$

$$dU = -G\,\frac{m\rho\, r^2\sin\theta\, dr\, d\theta\, d\phi}{(r^2 - 2rd\cos\theta + d^2)^{1/2}}$$

$$U = -G\, m\rho \int_0^R \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} \frac{r^2\sin\theta\, dr\, d\theta\, d\phi}{(r^2 - 2rd\cos\theta + d^2)^{1/2}}$$

$\theta \qquad \phi$

$$\frac{d}{ds} f(s) = \frac{d\theta}{ds}\frac{d}{d\theta} f(s)$$

$$\frac{d}{ds} s^2 = \frac{d}{ds}(r^2 - 2rd\cos\theta + d^2)$$

$$\frac{d\theta}{ds}\frac{d}{d\theta}$$

$$2s = \frac{d\theta}{ds}\cdot(2rd\sin\theta)$$

$$2s\,ds = \underbrace{2rd\sin\theta}\, d\theta \longrightarrow \frac{s\,ds}{d} = r\sin\theta\, d\theta$$

$$U = -\frac{G\, m\rho}{d} \iiint \frac{r\,\cancel{s}\, ds\, dr\, d\phi}{\cancel{s}}$$

$$= -\frac{G\, m\rho 2\pi}{d} \int_0^R r\,dr \int_{-r+d}^{r+d} ds$$

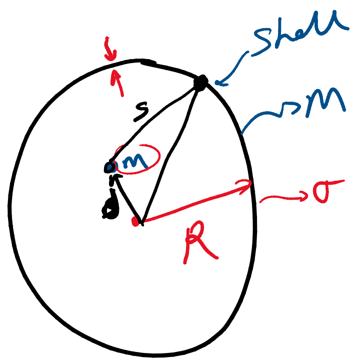
$$= -\frac{G m\rho 2\pi}{d} \int_0^R \underbrace{r\,dr.\,(r+d - (-r+d))}_{2r^2 dr}$$

$$= -\frac{G m\rho 2\pi}{\ldots}\ \frac{2}{3} R^3 = -G\, m\, \underbrace{\rho\cdot\frac{4\pi}{3}R^3}_{M}$$

$$= -\frac{Gm\rho\, 2\pi}{d}\, 2 \cdot \frac{R^3}{3} = -G\frac{m\left(\rho \cdot \frac{4\pi}{3} R^3\right)}{d}$$

$$= -G\frac{mM}{d}$$

⇒ Potential inside a spherical shell

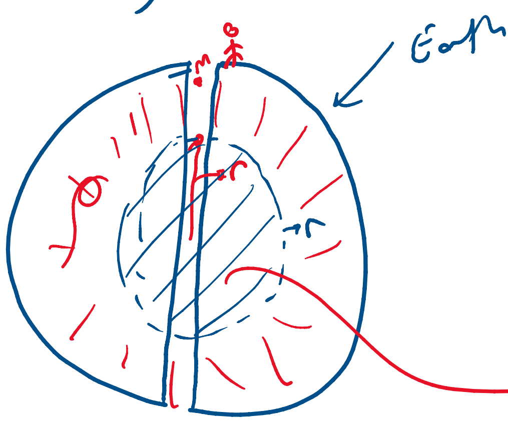


$$U = -G\frac{m\left((\sigma . dr) . 2\pi R\right)}{2dR} \int_{R-d}^{R+d} ds$$

$$\boxed{U = -G\frac{mM}{R}}$$

$$\nabla U = 0$$

⇒ Journey through the center of the earth



~12 km

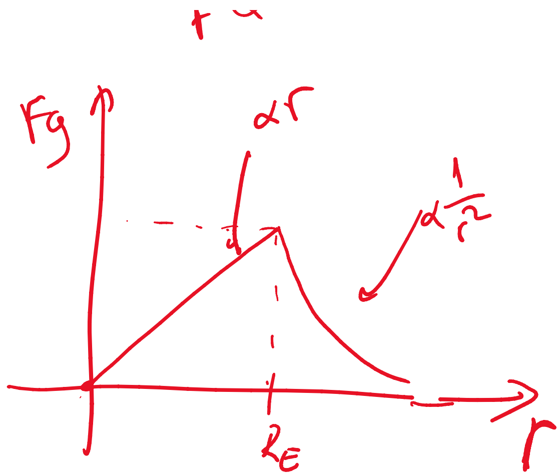
Russia

$$f_g = ?$$

$$m_r = \frac{V_r}{V_E} \cdot M_E \qquad m_r = M_E \cdot \frac{r^3}{R_E^3}$$

$$f \propto x$$

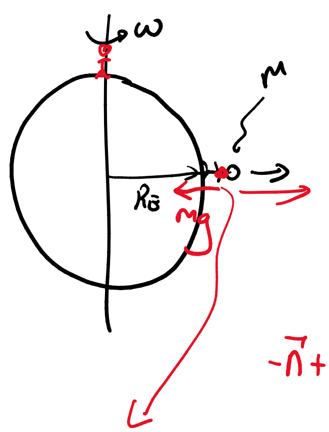
$$f_g = G\frac{Mm}{r^2} = G\frac{m}{r^2} \cdot \left(M_E \frac{r^3}{R_E^3}\right)$$



$$F_g = G\frac{Mm}{r^2} = G\frac{m}{r^2}\cdot\left(M_E\frac{r^3}{R_E^3}\right)$$

$$\boxed{F_g = G\frac{mM_E}{R_E^3}\cdot r}$$

⇒ Apperent weight & the earth's rotation



$$m\frac{v^2}{R_E} = m\omega^2 R_E$$

$$W_0 = mg$$

$$-\vec{n} + m\vec{g} = m\omega^2 \vec{R}_E$$

$$\boxed{W = mg - m\omega^2 R_E}$$

apparent weigh

$\sim 9.8\ m/s^2$

$0.0339\ m/s^2$

$W \to g_e = 9.78\ldots$

$g_p = 9.81\ldots$