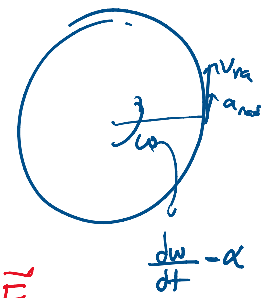
Lecture 20

2 Aralık 2019 Pazartesi 10:40

Previos week: **Rotational Motion**

$\omega \rightarrow V_{rad} = R\omega$

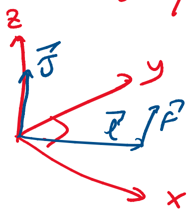
$\alpha \qquad a_{rad} = R\cdot\alpha$



$I \rightarrow$ moment of inertia

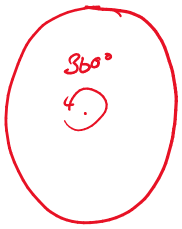
$\sum_i m_i r_i^2$

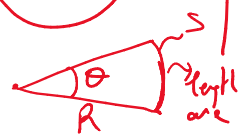
Torque $\Rightarrow \vec{\tau} = \vec{\ell} \times \vec{F}$



$|\vec{\tau}| = F\ell\sin\theta$

$= I\alpha$





$S = R\theta$ — 360° (radian)

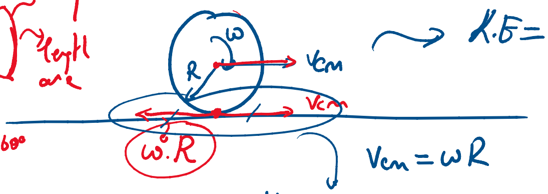
$S = 2\pi R = R\theta$

radians – degrees

$\frac{360°}{2\pi}$

↳ Combined Kinetic Energy:

$K.E = \frac{1}{2}I\omega^2 + \frac{1}{2}mV_{cm}^2$



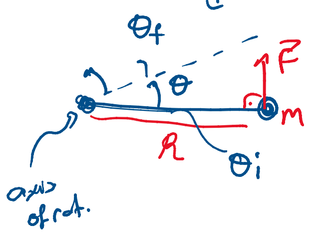
$V_{cm} = \omega R$

> Rolling without slipping: → bottom point of the rolling object is still

$\omega\cdot R = V_{cm}$

> Work: $W = \int_{\theta_1}^{\theta_f} \tau_z \, d\theta = \Delta K_{rot}$

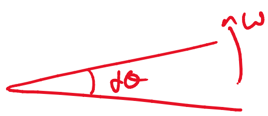
Torque

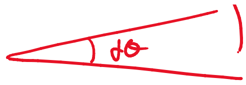


$\tau = F\cdot R$

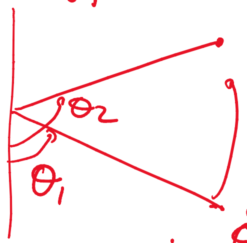
Midterm II — 14th of Dec

Angular Momentum





$$\frac{d\theta}{dt}=\omega$$



250 rad

16 rad

$$\omega_{av}=\frac{\theta_2-\theta_1}{\Delta t}=78\ \text{rad/s}$$

$\Delta t \to 3.0\,s$

rev/s

$/2\pi$ → rev/s

$$\frac{d\theta}{dt}=\lim_{\Delta t\to 0}\frac{\theta_2-\theta_1}{\Delta t}$$

1 rev → rad?

$2\pi\ \text{rad} = 1\ \text{rev}$

Angular Momentum

$$\vec F=\frac{d}{dt}\vec p \Longrightarrow \vec\tau=\vec\ell\times\vec F=\vec\ell\times\frac{d\vec p}{dt}$$

$$\vec\tau=\frac{d}{dt}(\vec\ell\times\vec p)=\underbrace{\frac{d\vec\ell}{dt}\times\vec p}_{\vec v\times m\vec v\ \to\ 0}+\vec\ell\times\frac{d\vec p}{dt}$$

$$\vec\tau=\frac{d}{dt}(\vec\ell\times\vec p)=\vec\ell\times\frac{d\vec p}{dt}=\vec\ell\times\vec F$$

$$\boxed{\vec L=\vec\ell\times\vec p}$$ angular momentum

all the conservation laws that we used for linear momentum applies here as well.

$$\vec L = I\vec\omega$$

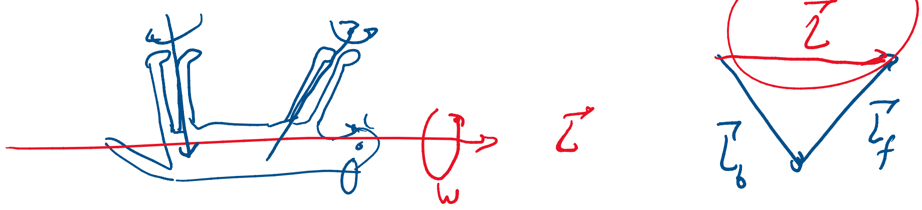
$\vec\omega$: angular

moment of Inertia — angular velocity

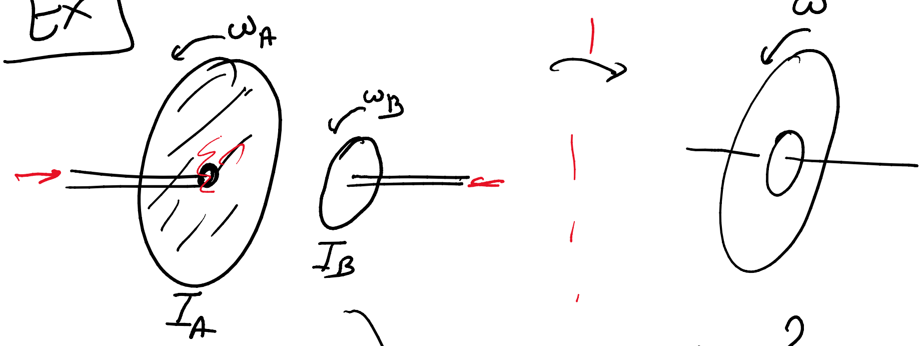
for linear momentum applies here as well.

When there are no external torque acting on an object $\frac{d\vec{L}}{dt} = 0$

→ How do the cats flip while falling down?



Ex



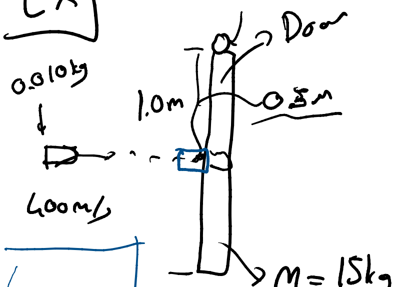
$\omega = ?$

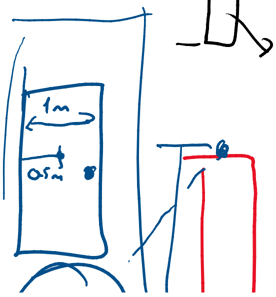
$$L_A = I_A \omega_A \qquad L_B = I_B \omega_B \quad > \quad L_A + L_B = (I_A + I_B)\,\omega$$

$$I_A \omega_A + I_B \omega_B = (I_A + I_D)\,\omega$$

$$\boxed{\omega = \frac{I_A \omega_A + I_B \omega_B}{(I_A + I_B)}}$$

Ex





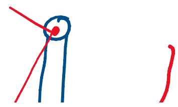
$\omega = ?$

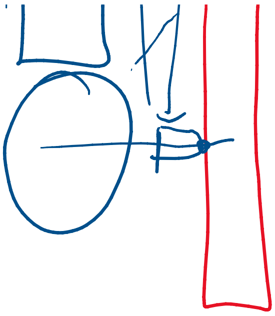
$L_i = 0$

$L_f = 0$

$$L_i = \underbrace{\left(0.010\,kg \cdot (0.5\,m)^2\right)}_{I_B} \cdot \left(\frac{400\,m/s}{0.5\,m}\right)$$

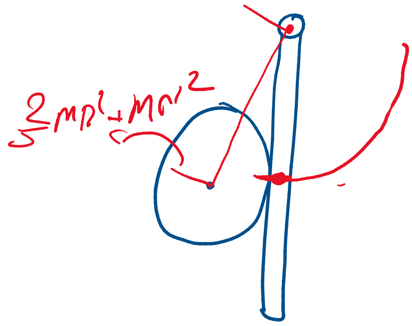
$$= 2.0\ kg\,m^2/s$$





$$= 2.0\ kg\,m^2/s$$

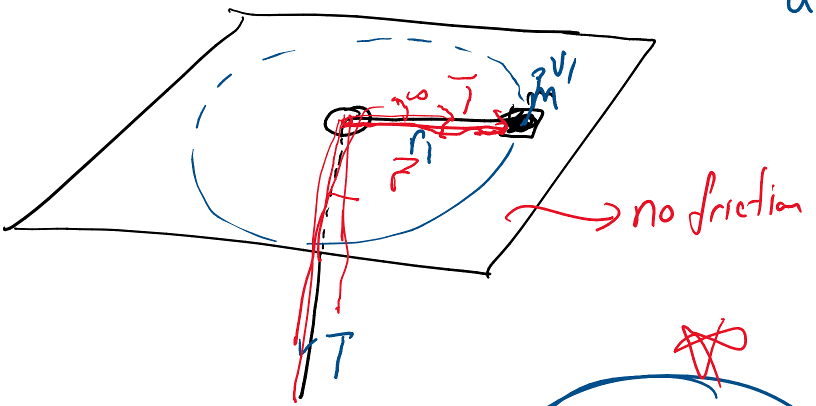
$$L_f = 2.0\ kg\,m^2/s = \underbrace{I}_{\frac{1}{3}M d^2 + I_B} \cdot \omega$$



$$\omega = \frac{2.0\ kg\,m^2/s}{\frac{1}{3}M d^2 + I_B} = 0.40\ rad/s$$

Ex 10.91

a) $r_1 \rightarrow r_2$ what would be the $T(r)$?



$$m\,\omega_{r_1}^2 r_1 = T(r_1)$$

$$m\omega^2 r = T(r)$$

$$L_{r_1} = L_r$$

$$\cancel{m} r_1^2 \cdot \omega_1 = \cancel{m} r^2 \omega$$

$$r^2 = \frac{r_1^2 \omega_1}{\omega}$$

$$\boxed{\omega = \frac{r_1^2}{r^2}\omega_1}$$

$$\frac{L_{r_1}}{I_{r_1}} = \frac{L_r}{I_r}$$

$$(m r_1^2)$$

$$m \frac{r_1^4}{r^4}\omega_1^2 \cdot r$$

$$\omega_1 \rightarrow v_1/r_1$$

$$\boxed{T(r) = m\frac{v_1^2 r_1^2}{r^3}}$$

$$\omega = \int_{r_1}^{r_2}$$

$$\omega = v/2r$$

$$\omega = v/2r$$

$$\frac{1}{2}I\omega^2 \quad \frac{1}{2}I\omega^2 - \ldots$$

$$\omega_1 \cdot r_1 = v_1$$

$$T(r_1) = m\underbrace{\omega_{r_1}^2}_{(v_1/r_1)^2} r_1$$

$$T(r) = m\omega^2 r$$