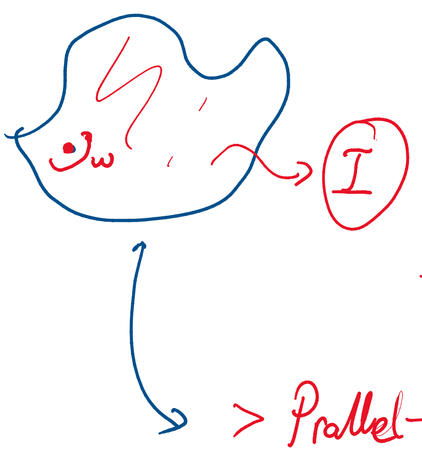
Lecture 19

28 Kasım 2019 Perşembe 08:42

Previous lecture: we defined moment of inertia:

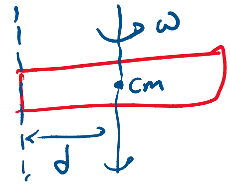


$$I = \sum_i m_i r^2$$

$\frac{d\vec{\omega}}{dt}$ ($\vec{\omega}$ → angular velocity ⟷ $V_{tan} = R\omega$

$\vec{\alpha}$ → angular acceleration $a_{tan} = R.\alpha$

> Parallel-axis theorem



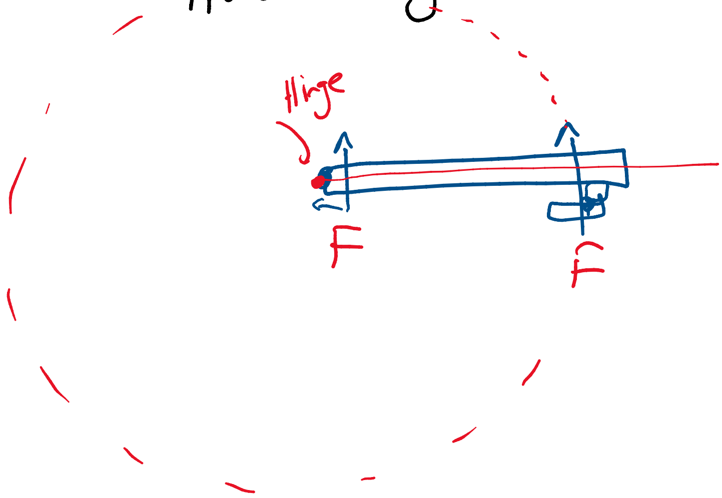
I_2 I_{cm}

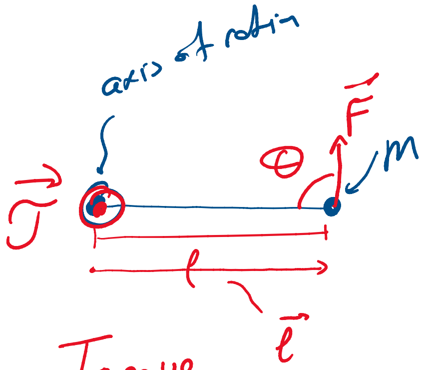
$$I_2 = I_{cm} + Md^2$$

$$K.E. = \frac{1}{2} I \omega^2$$

Chapter 10 – Dynamics of Rotational Motion

How do you rotate an object?





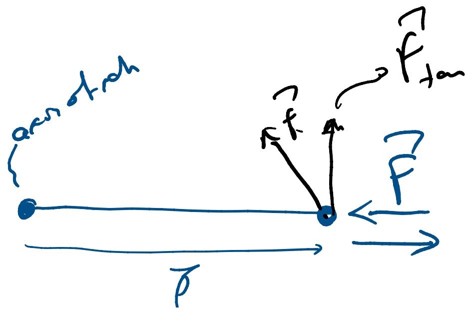
Torque

$$\vec{\tau} = \vec{l} \times \vec{F}$$

$$\tau = F.l\sin\theta$$

×

⊙



τ

$$\tau = F \cdot l \sin\theta = l \cdot F_{tan}$$

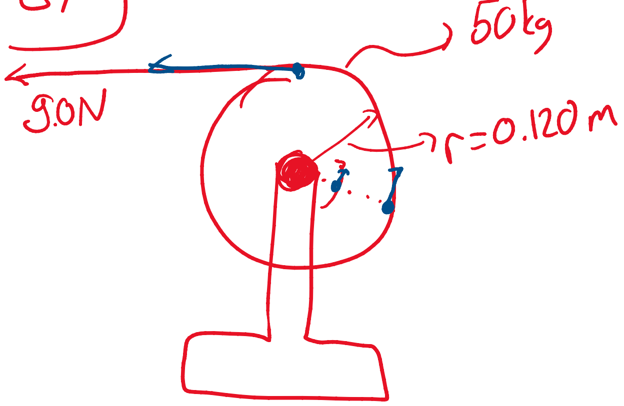
$F_{tan} = m \cdot a$, $a = \alpha \cdot l$

$$= m l^2 \alpha$$

$m l^2 = I$

$$\tau = I \cdot \alpha \qquad \vec{\tau} = I \vec{\alpha}$$

Ex)



What is the acceleration of the rope?

$$\tau = 9.0N \cdot 0.120m$$
$$= 1.08 N.m$$

$$V = \omega \cdot r$$

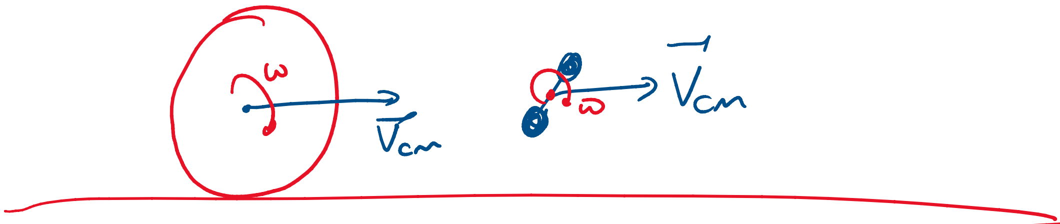
$$\tau = I \cdot \alpha = \frac{1}{2} m \cdot r^2 \cdot \alpha$$

$$1.08 N.m = 0.36 kgm^2 \cdot \alpha$$

$$\alpha = 3 \, rad/s^2$$

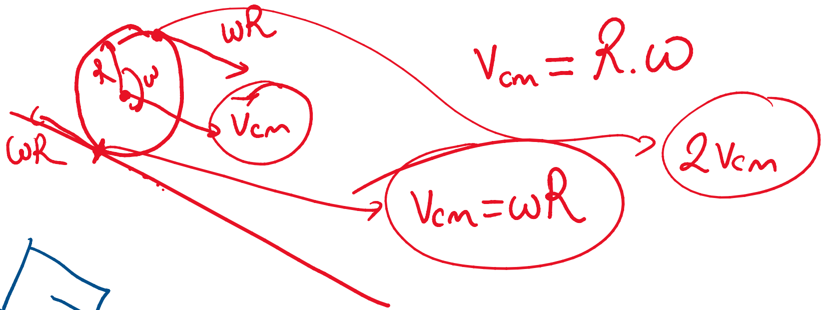
$$a_{rope} = \alpha \cdot r$$
$$= 3 \, rad/s^2 \cdot 0.120m$$
$$a_{rop} = 0.36 \, m/s^2$$

Combined Translation & Rotation

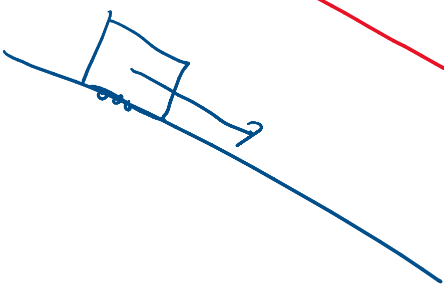


$$K.E. = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \omega^2$$

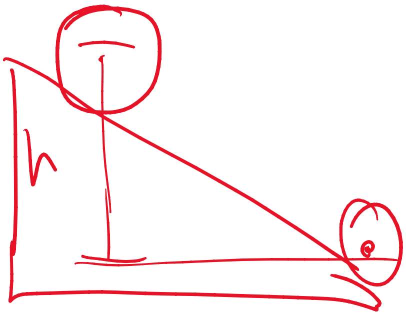
↳ Rolling w/o slipping



$V_{cm} = R.\omega$



Ex)



Sphere, hollow sphere, hollow cylinder
rod

which one will reach the bottom faster assuming same mass & radius? (roll w/o slipping)

$$mgh = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \omega^2$$

$\omega^2 = \left(\frac{V_{cm}}{R}\right)^2$

$I = c M.R^2$

$I_{sph} = \frac{2}{5} MR^2$

$I_{HS} = \frac{2}{3} MR^2$

$I = c\,M.R$

$I_{HS} = \frac{2}{3} \ldots$

$I_{HC} = (1) mR^2$

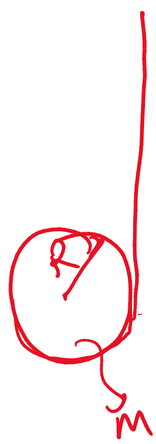
$I_{Rd} = (\frac{1}{2}) mR^2$

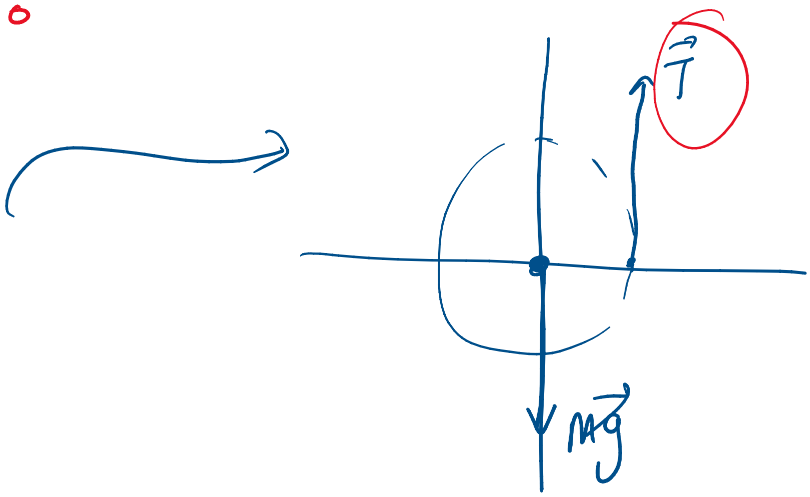
$$mgh = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} c\, mR^2 \frac{v_{cm}^2}{R^2}$$

$$v_{cm} = \sqrt{\frac{2gh}{1+c}}$$

→ Sphere > Rod > Hollow sph > Hollo Cylinder

Ex) Yo-Yo





$$mg - T = ma_{cm}$$

$$\tau = T.R = I.(a_{cm}/R)$$

$$T = \frac{1}{2} mR^2 \frac{a_{cm}}{R^2}$$

$$mg = \frac{1}{2} ma_{cm} + ma_{cm}$$

$$a_{cm} = \frac{2g}{3}$$

$$a_{cm} = \frac{2g}{3}$$