Lecture 10

24 Ekim 2019 Perşembe 09:38

> We are working on a more abstract way to study dynamics

$\vec{F}(\vec{r})$ →

$\vec{F} \propto \frac{1}{r^2}$

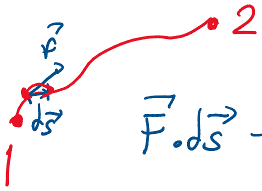
→ Gravity

Electromagnetic force

↳ Electric, magnetic

> Work & Kinetic Energy

$$dW = \vec{F} \cdot d\vec{s}$$



$\vec{F} \cdot d\vec{s}$ → scalar quantity
Work

$$W_{12} = \int_1^2 \vec{F} \cdot d\vec{s}$$

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$d\vec{s} = \frac{d\vec{s}}{dt}dt = \vec{v}\,dt$$

Work ⟺ Kinetic Energy

"Energy of an object in motion"

$$\int m \frac{d\vec{v}}{dt} \cdot \vec{v}\,dt \quad (m \text{ constant})$$

$$\frac{d\vec{v}}{dt} \cdot \vec{v} = \frac{1}{2}\frac{d}{dt}(\vec{v} \cdot \vec{v})$$

$$\int \frac{d}{dt}\left(\frac{1}{2}m v^2\right) dt$$

$$= \int_{v_1}^{v_2} d\left(\frac{1}{2}mv^2\right) = \frac{1}{2}m\left(v_2^2 - v_1^2\right) = W_{12}$$

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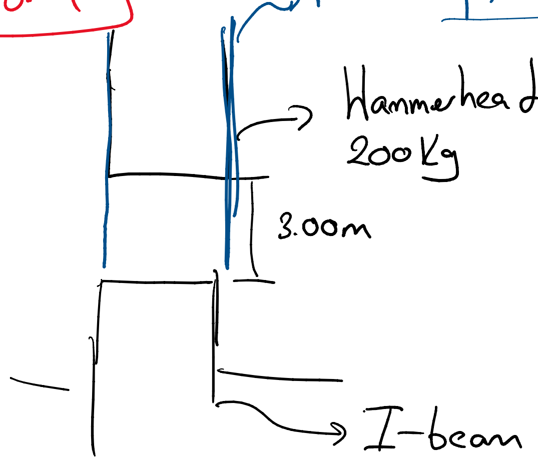
(v_2: final velocity, v_1: initial velocity)

$$\boxed{\Delta K = W_{Tot}}$$

Ex 6.4)

rails → force of friction 60N



Hammerhead, drives beam into ground by 7.4cm in every hit.

a) speed of the Hammerhead just before the contact?

b) Force applied by the HH on the beam?

a)

$\vec{f}_f = 60N$

$\hat{x} \downarrow$ $\quad \vec{\omega} = 200kg \times 9.8 m/s^2$

$$\boxed{\vec{\omega} - \vec{f} = \sum \vec{F} = m\vec{a}}$$

$$|\vec{\omega} - \vec{f}| = 1900\,N = ma$$

$$\frac{1900N}{200kg} = 9.5\,m/s^2$$

$$v^2 = 2 . a . x$$

(x = 3.00 m, a = 9.5 m/s²)

$$\boxed{v = 7.55\,m/s}$$

or

$$1900\,N\hat{x} \cdot 3.00\,m\hat{x} = 5700\,J = \Delta K = \frac{1}{2}mv^2 - 0$$

(initial velocity was 0)

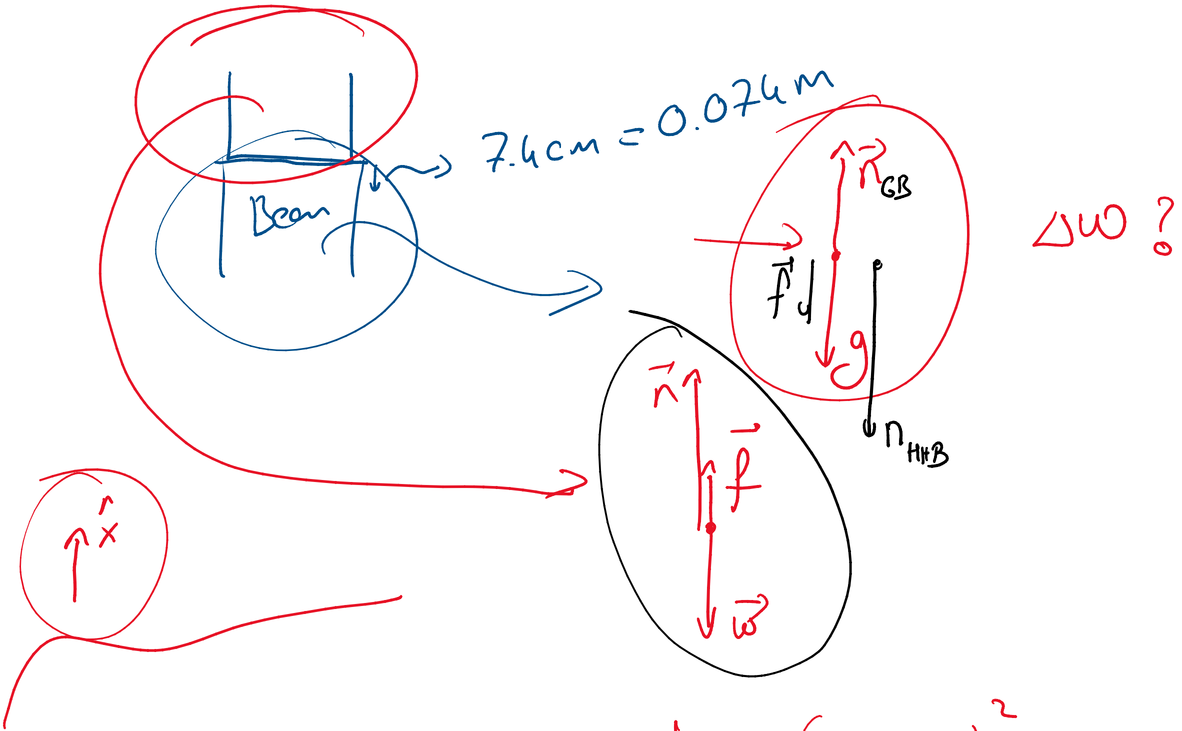
$$= 100\,kg\, v^2$$

$$\sqrt{57} = v$$

$$\boxed{v = 7.55\,m/s}$$

$v = 7.55\,m/s$

b) ~~Work done by~~ hammerhead on the beam force



$$(\vec{n}+\vec{f}-\vec{w})\cdot 0.074\,m(\hat{x}) = 0 - \frac{1}{2}200\,kg\,(7.55\,m/s)^2$$

$\vec{f}$: 60N, $\vec{w}$: 1960N; right-hand side: $-5700\,J$

$n = 79000\,N$ → 8 ton of mass

⇒ Work & Energy with Varying Forces

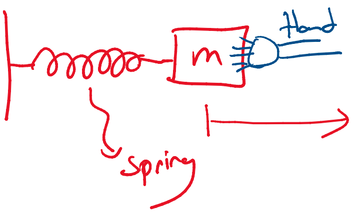
$$W = \int_{\vec{s}_1}^{\vec{s}_2} \vec{F}\cdot d\vec{s}$$

$\vec{F} \to \vec{F}(\vec{s})$

Ex)

F(s)

Ex)

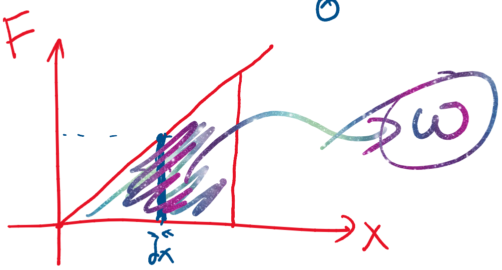


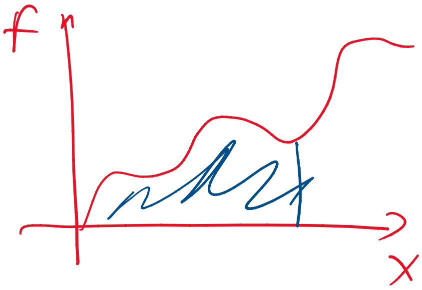
$F = kx$ (Hooke's Law)

spring constant

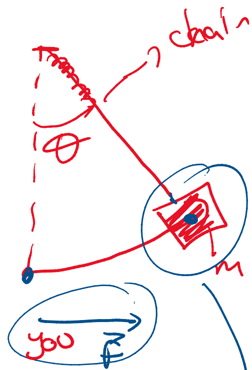
Work done by the hand?

$$W = \int_0^x \underbrace{kx'\hat{x}}_{\vec{F}} \cdot \underbrace{\hat{x}\,dx'}_{d\vec{s}} = \int_0^x kx'\,dx' = \frac{1}{2}kx^2$$





Ex)

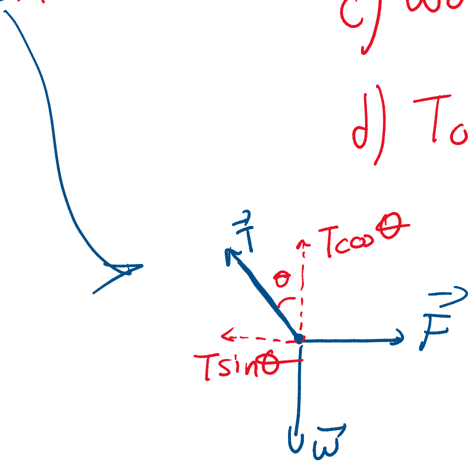


a) Work done by you?

b) Work done by chain?

c) Work done by gravity?

d) Total work done by all forces?

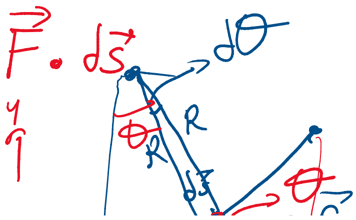


$F = T\sin\theta$

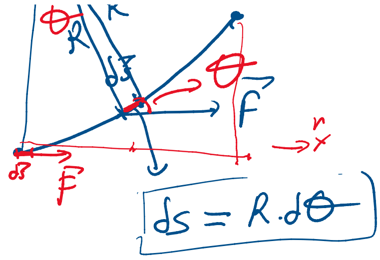
$mg = T\cos\theta$

$F = mg\tan\theta$ ($\frac{\sin\theta}{\cos\theta}$)

a) W_y



ii)



$$d\vec{s} = \hat{x}\,R\,d\theta\cdot\cos\theta + \hat{y}\,R\,d\theta\,\sin\theta$$

$$\vec{F}\cdot d\vec{s} = mg\tan\theta\,\hat{x}\cdot\left(\hat{x}\,R\,d\theta\cos\theta + \hat{y}\,R\,d\theta\sin\theta\right)$$

$$= mg\tan\theta\cos\theta\,R\,d\theta$$

$$= mg\sin\theta\,R\,d\theta$$

$$\int_0^{\theta_f} mgR\sin\theta\,d\theta = -mgR\cos\theta\Big|_0^{\theta_f}$$

$$= mgR\left(1-\cos\theta_f\right)$$