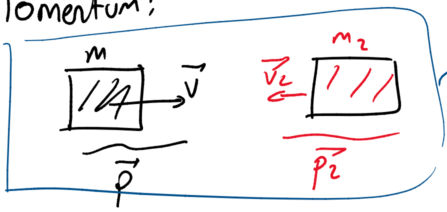
Lecture 18

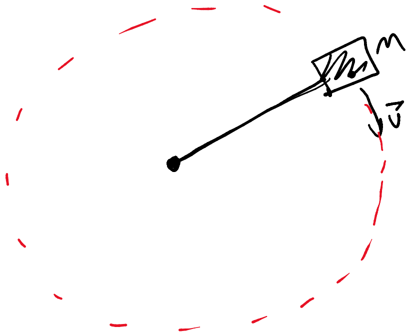
25 Kasım 2019 Pazartesi 10:41

> Previous week:

Momentum:

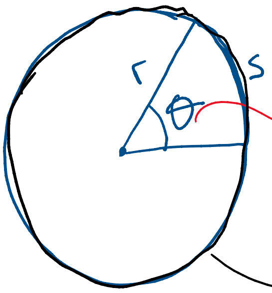


If no external forces, the momentum is conserved.



momentum for the rotating object?

Chapter 9- Rotation of Rigid Bodies



degrees 360°

$$\boxed{\theta = \frac{s}{r}}$$

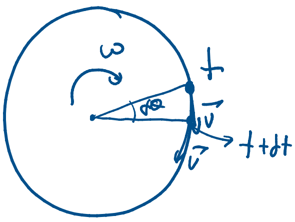
radian

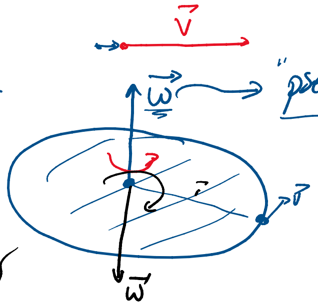
$$\boxed{1\,\text{rad} = \frac{360^\circ}{2\pi} = 57.3^\circ}$$

$$\frac{s}{r} = 2\pi \rightarrow \boxed{s = 2\pi r}$$

- Angular velocity: $\omega = \frac{d\theta}{dt}$

"vector quantity"



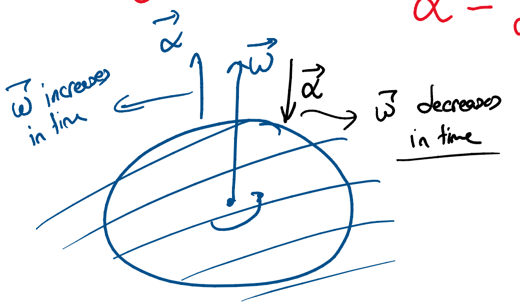


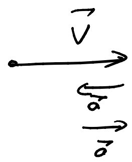
direction of rotation follows a right-hand rule

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

○ Angular acceleration;

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}$$



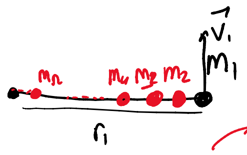


↳ Constant angular acceleration;

$$\boxed{\omega_z = \omega_{0z} + \alpha_z t} \quad \boxed{\theta = \theta_0 + \omega_{0z} t + \frac{1}{2}\alpha_z t^2}$$

$$\boxed{\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)}$$

⇒ Energy in Rotational Motion



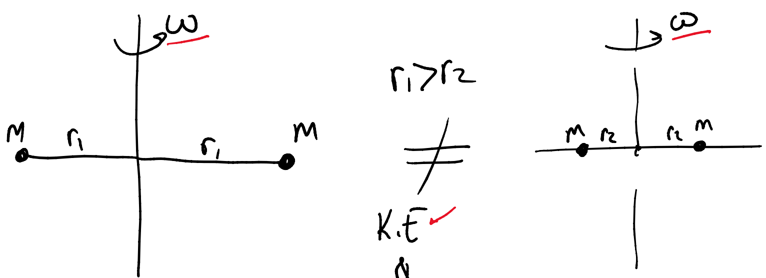
$$K.E = \frac{1}{2} m_1 \overset{r_1\omega}{v_1^2} = \frac{1}{2} m_1 r_1^2 \omega^2$$

$$K.E = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 + \dots$$

$$\frac{1}{2} m_n r_n^2 \omega^2 + \dots \frac{1}{2} m_N r_N^2 \omega^2$$

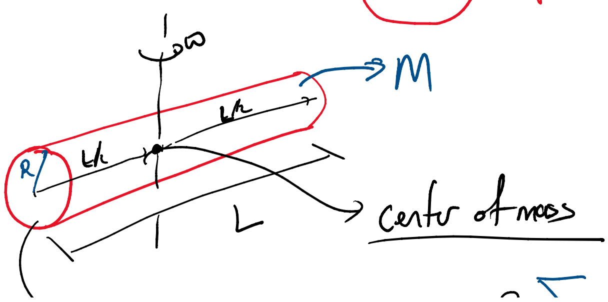
$$K.E = \frac{1}{2}\left(\underbrace{\sum_{i=1}^{N} m_i r_i^2}_{I}\right)\omega^2$$

I "moment of inertia"



I independent of angular velocity.

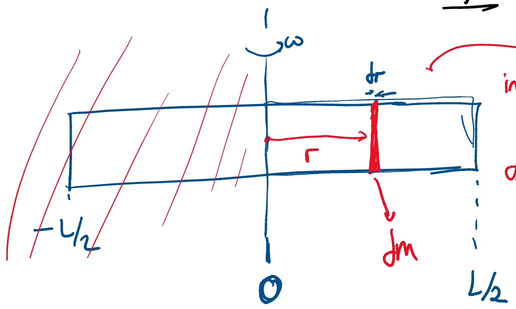
⇒



I L → center of mass

$$I_o = ?\sum m_i r_i^2$$

integration $\int$ infinitesimal mass



$$dI = dm \cdot r^2$$

$$dm = (dr \cdot \cancel{\pi R^2}) \cdot \rho$$

density of the rod

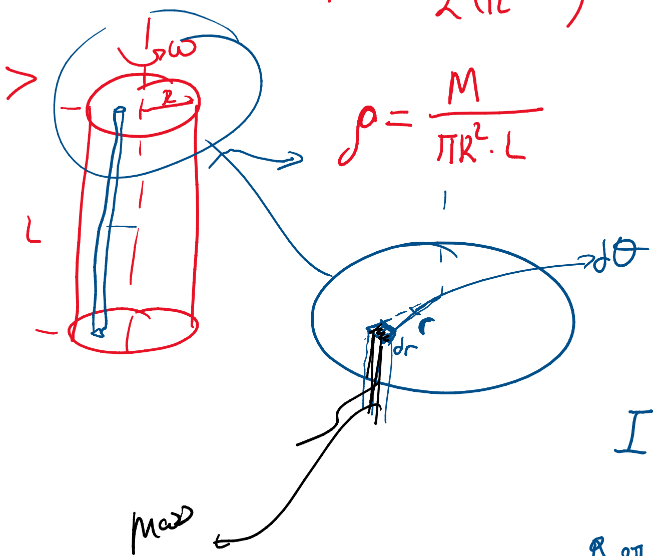
$$\rho = \frac{M}{\cancel{\pi R^2} \cdot L}$$

$L \to L/2$

$$I = \int_{0 \to -L/2} \frac{M}{L} \cdot r^2 \cdot dr$$

$$I = \frac{M}{L}\frac{r^3}{3}\Big|_{-L/2}^{L/2} = \frac{1}{12} M L^2$$

$$K.E = \frac{1}{2}\left(\frac{1}{12} M L^2\right)\omega^2$$



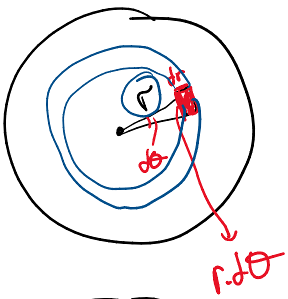
$$\rho = \frac{M}{\pi R^2 \cdot L}$$

$$r d\theta \cdot dr \cdot L$$

$$I = \int dm\, r^2$$

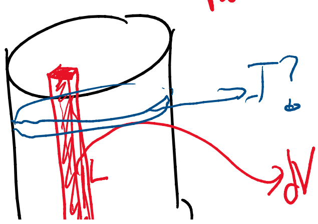
$$\rho \cdot r d\theta \cdot dr \cdot L$$

$$I = \int_0^R \int_0^{2\pi} \frac{M}{\pi R^2 \cancel{L}} \cdot \cancel{L}\, r^3 dr\, d\theta$$

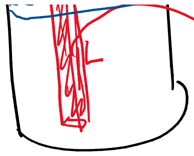


$$I = \int_0^R \frac{M}{\cancel{\pi} R^2} r^3 dr \cdot 2\cancel{\pi} = \int_0^R \frac{2M}{R^2} r^3 dr$$

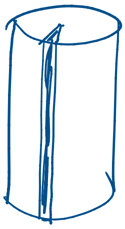
$$= \frac{2M}{R^2}\frac{r^4}{4}\Big|_0^R = \frac{1}{2} M R^2$$

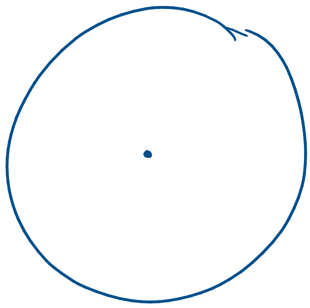


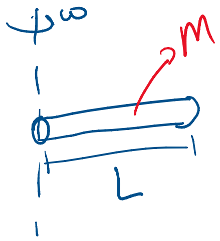
$$dV = L \cdot r d\theta dr$$



$dV = L \cdot r\, d\theta\, dr$

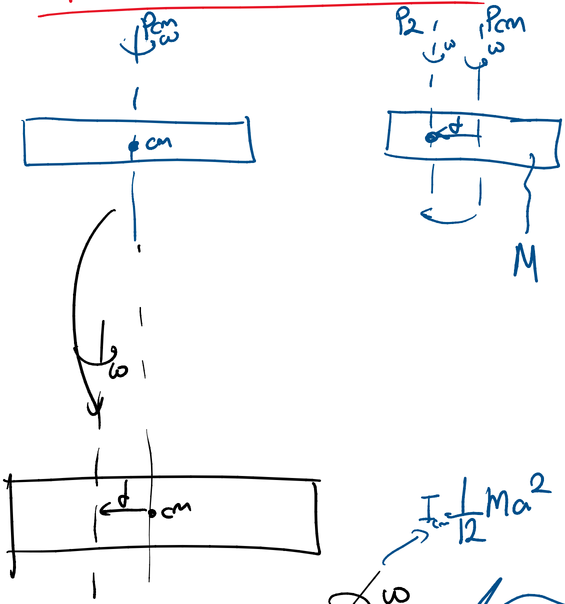






$I = \frac{1}{3} M L^2$

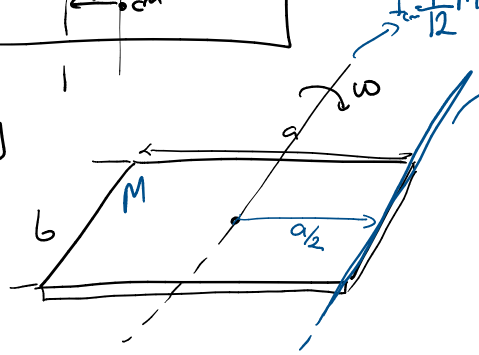
↳ Parallel-axis theorem



$P_{cm} \,//\, P_2$

$$\boxed{I_{P_2} = I_{cm} + \underbrace{M}_{\text{Total mass of the object.}} d^2}$$

Ex)

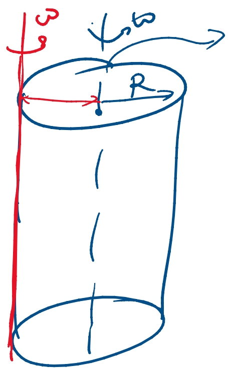


$I_{cm} = \frac{1}{12} M a^2$

$I_2 = ?$

$$I_2 = \frac{1}{12} M a^2 + M \frac{a^2}{4}$$

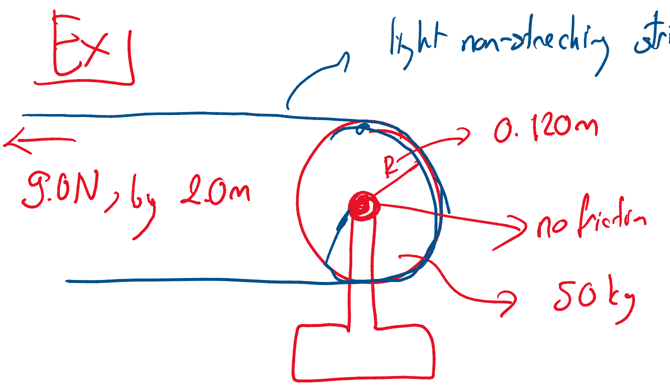
$$\boxed{I_2 = \frac{1}{3} M a^2}$$



$\frac{1}{2} M R^2$

$$\frac{1}{2} M R^2 + M R^2 = \underbrace{\frac{3}{2} M R^2}_{\text{new } I}$$

Ex



find the angular speed and the final speed of the cable!

$$W = \Delta K$$

$$9.0N \cdot 2.0m = 18J = \frac{1}{2} I \omega^2$$

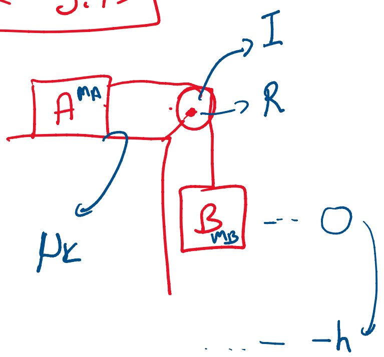
$$I = \underbrace{\left(\frac{1}{2} R^2 m\right)}_{\left(\frac{1}{2}\, 0.120m^2 \cdot 50kg\right) = 0.36\, kgm^2}$$

$$\omega^2 = 160\ rad^2/s^2$$

$$\boxed{\omega = 10\ rad/s}$$

$$\boxed{v = \omega . R = 10\, rad/s \cdot 0.120m = 1.2m/s}$$

Ex - 9.75



$v_B = ?$

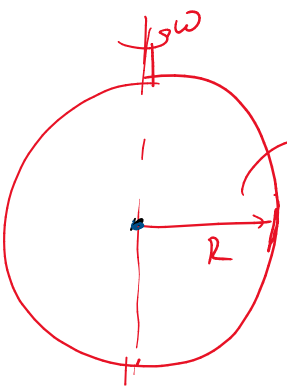
$$m_B g h = \frac{1}{2} m_B v_B^2 + \frac{1}{2} m_A v_B^2 + m_A g \cdot \mu_k \cdot h + \frac{1}{2} I \omega^2$$

$$\omega = \frac{v_B}{R}$$

$$m_B g h - m_A g h \mu_k = \frac{1}{2}\left(m_B + m_A + \frac{I}{R^2}\right) v_B^2$$

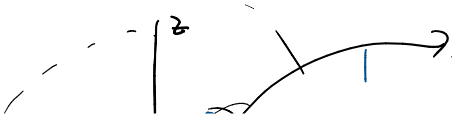
$$v_B = \sqrt{\frac{2gh(m_B - \mu_k m_A)}{(m_B + m_A + I/R^2)}}$$

Ex

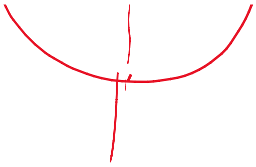


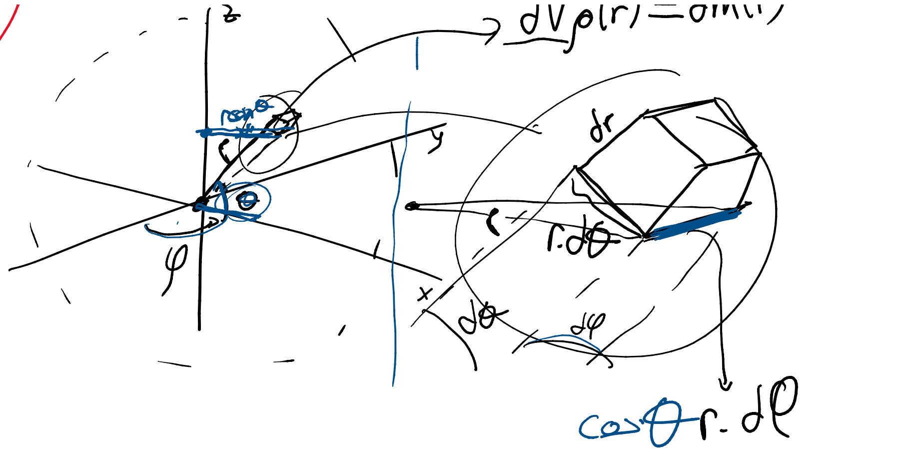
$\rho(r) = a - br$ (density), $a > Rb$

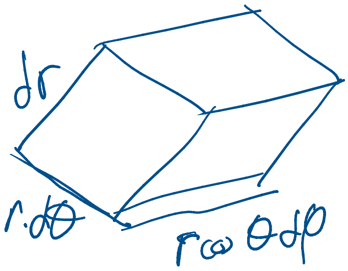
$I = ?$



$$dV \rho(r) = dm(r)$$







$$\boxed{dV = r^2 \cos\theta \, d\theta \, d\varphi \, dr}$$

$$dm = dV . \rho(r)$$

$$I = \int dm . r^2 = \int (a - br) . r^4 \cos\theta \, d\theta \, d\varphi \, dr$$

$$= \int_0^R \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} (a - br) \, r^4 dr \cos\theta \, d\theta \, d\varphi$$

$$= \int_0^R \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} \left(a r^4 \cos\theta - b r^5 \cos\theta \right) dr \, d\theta \, d\varphi$$

$$= \int_0^R \int_{-\pi/2}^{\pi/2} 2\pi \left(a r^4 \cos\theta - b r^5 \cos\theta \right) dr \, d\theta$$

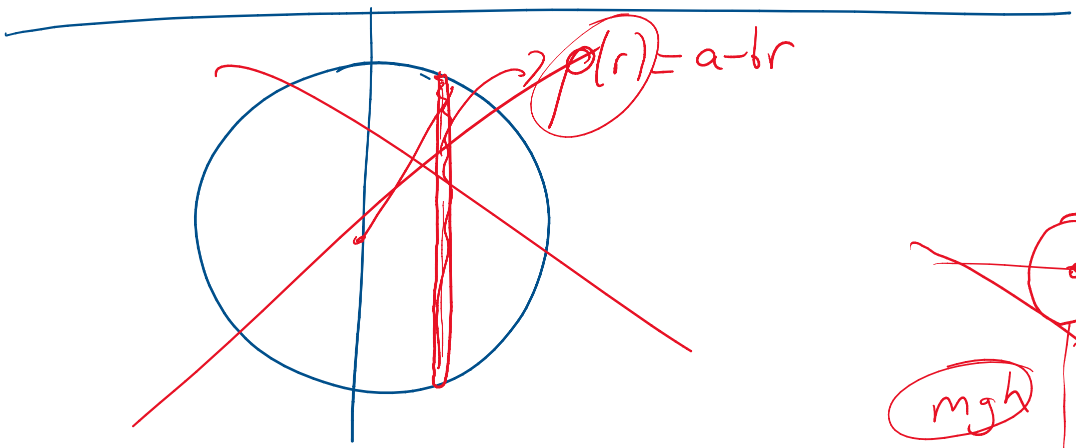
$$= \int_0^R 2\pi \left[a r^4 \underbrace{\left(\sin\theta \Big|_{-\pi/2}^{\pi/2} \right)}_{2} - b r^5 \underbrace{\sin\theta \Big|_{-\pi/2}^{\pi/2}}_{2} \right] dr$$

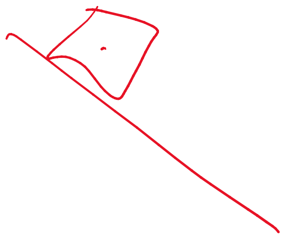
$$= \int_0^R 2\pi \left[2 a r^4 - 2 b r^5 \right] dr$$

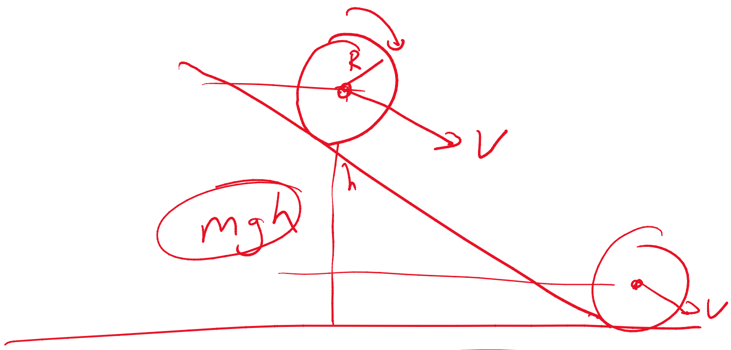
$$= \int_0 2\pi [2ar - 2br] ...$$

$$= 4\pi a \frac{r^5}{5}\Big|_0^R - 4\pi b \frac{r^6}{6}\Big|_0^R$$

$$\boxed{I = \frac{4\pi a R^5}{5} - \frac{4\pi b R^6}{6}}$$







$$mgh = \boxed{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}$$

$$\frac{1}{2}MR^2 \cdot \frac{v^2}{R^2}$$

$$\cancel{m}gh = \cancel{m}v^2 \qquad v = \sqrt{gh}$$