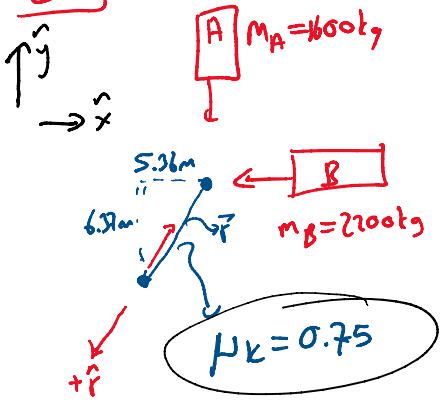
Lecture 17

21 Kasım 2019 Perşembe 09:40

Ex)



→ How fast was each car travelling just before the collision?

$$\vec{F}_f \cdot \vec{r} = W_f = \Delta K$$

$$-\left(\mu_k (2200kg + 1600kg) \cdot 9.8 m/s^2\right) \hat{r} \cdot \hat{r} \cdot |r|$$

$$W_f = -232519\ J = 0J - \left(\frac{1}{2} 3800kg \cdot v_i^2\right)$$

$$v_i = 11.1\ m/s$$

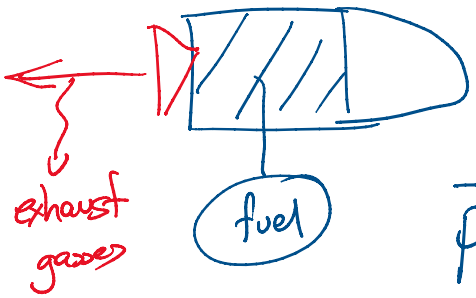
$\hat{x}, \hat{y}$

$$\vec{p}_A + \vec{p}_B = \vec{p}_{AB}$$

$-\hat{y}$ $-\hat{x}$ $-\hat{x}, -\hat{y}$

⇒ Rocket Propulsion

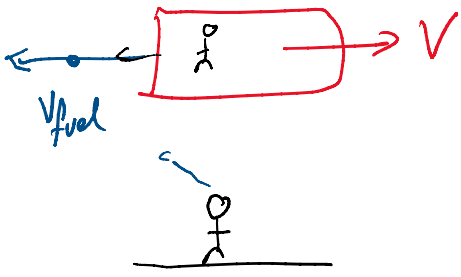
free space; no gravity
no air



$$\vec{p} = m\vec{v}$$

m: variable

time t



$$v_{fuel} = V - v_{ex}$$

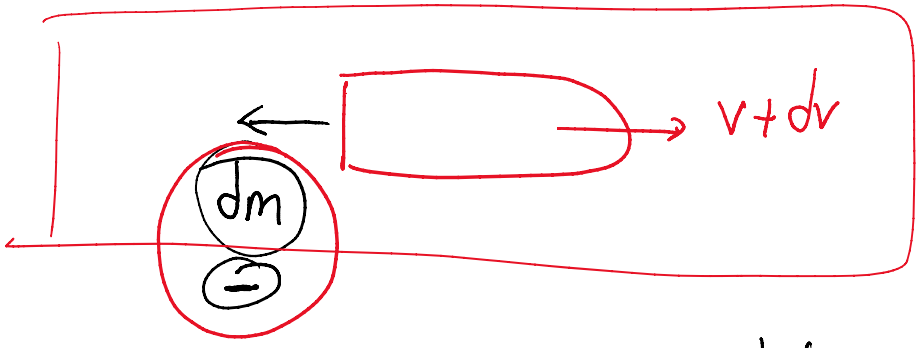
v_{ex}: Velocity of exhaust gas with respect to

$dm = m_f - m_i$

exhaust gas with respect to the rocket

time $t+dt$

dm



$\Rightarrow$ momentum of the rocket:

$$= (m+dm)\cdot(v+dv)$$

$\Rightarrow$ Momentum of the fuel ejected:

$$= -dm(v-v_{ex})$$

$$P_2 = (m+dm)(v+dv) + (-dm)(v-v_{ex})$$

$$= mv + mdv + vdm + dmdv - vdm + v_{ex}dm$$

$$= mv + mdv + v_{ex}dm + dmdv$$

$\vec{p}(t)$

$P_1 = P_2$

mv

$\vec{p}(t+dt)$

ignore

$$mv = mv + mdv + v_{ex}dm + dmdv$$

$m\,dv = -v_{ex}\,dm$

dt

constant

$$\star \quad \boxed{m\frac{dv}{dt} = -v_{ex}\frac{dm}{dt}}$$

rate at which the fuel is burnt

Force

speed the fuel is expelled

v_0

$$\int_{v_0}^{v} dv' = -v_{ex}\int_{m_0}^{m}\frac{dm'}{m'}$$

$$\boxed{v - v_0 = v_{ex}\ln\frac{m_0}{m}}$$

Ex) For the first second of a rocket engine: $\frac{m_0}{120}$ What is the acceleration of the rocket?

$v_{ex} = 2400\ m/s$

$$\frac{dm}{dt} = \frac{m_0/120}{1s} = \frac{m_0}{120s}$$

$$m_0 a = 2400\ m/s \cdot \frac{m_0}{120s}$$

$$a = \frac{2400\ m/s}{120s} = 20\ m/s^2$$

$$v = 2400\ m/s \ln\left(\frac{m_0}{m_0 - m_0/120}\right)$$