Lecture 16

18 Kasım 2019 Pazartesi 10:40

# Chapter 8 – Momentum, Impulse and Collisions

↳ We know that every object with velocity carries a certain kinetic energy

$$\hookrightarrow \Delta K = W = \vec{F} \cdot \vec{s}$$

$$m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v})$$

$$m\vec{v} = \vec{p} \quad \text{momentum}$$

$$\boxed{\vec{F} = \frac{d}{dt}\vec{p}}$$

## The Impulse-Momentum Theorem

$$\boxed{\vec{J}} = \sum \vec{F} \cdot \underline{\Delta t}$$

$\vec{J}$: Impulse of a constant net force

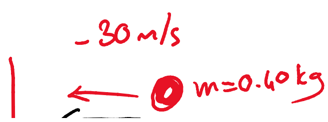
$\Delta t$: time interval over where net force acts

$$\vec{p}_2 - \vec{p}_1 = \Delta\vec{p}$$

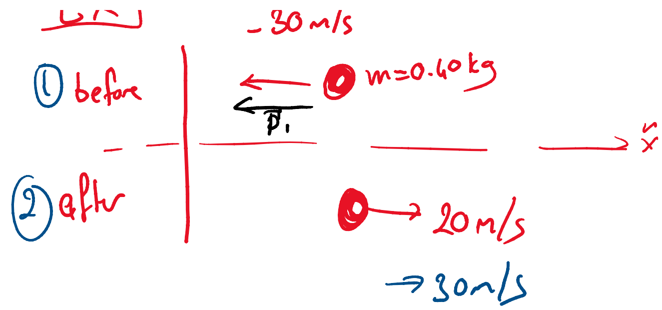
$$\lim_{\Delta t \to 0} \vec{J} = \int_{t_1}^{t_2} \sum \vec{F}\, dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{\cancel{dt}}\, \cancel{dt} = \int_{\vec{p}_1}^{\vec{p}_2} dp = \underline{\vec{p}_2 - \vec{p}_1}$$

Ex)

① before




a) Find the impulse during the collision




a) Find the impulse during the collision

b) If the contact lasted for 0.010 s find the force during the impact.

a) $\vec{P}_1 = -30\frac{m}{s} \cdot 0.40\,kg\,\hat{x} = -12\ kgm/s\ \hat{x}$

$\vec{p}_2 = 20\,m/s \cdot 0.40\,kg\,\hat{x} = 8\ kgm/s\ \hat{x}$

$\vec{p}_2{}' = 12\ kgm/s\ \hat{x}$ $\quad \vec{J} = 24\ kgm/s\ \hat{x}$

$\vec{J} = \vec{p}_2 - \vec{p}_1$

$= [8\ kgm/s - (-12\ kgm/s)]\hat{x}$

$= 20\ kgm/s\ \hat{x}$

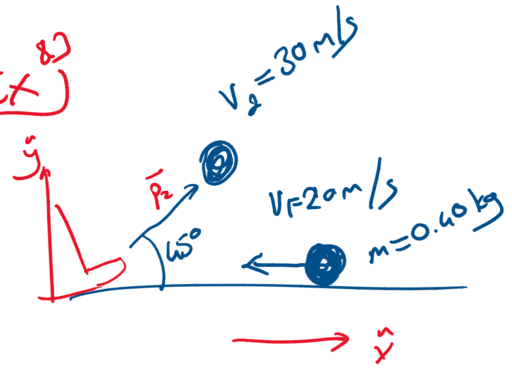
b) $$\vec{F} = \frac{\vec{J}}{\Delta t} = \frac{20\ kgm/s\ \hat{x}}{0.010\,s} = 2000\,N\,\hat{x}$$

⇒ How much energy does the ball lost during the collision?

$$K_1 - K_2 = \frac{1}{2}\,0.40\,kg\left((30\,m/s)^2 - (20\,m/s)^2\right)$$

$$= \underline{100\,J}$$

Ex 8)




$\Delta t = 0.010\,s$ $\quad \vec{J} = ?$

Average net $\vec{F} = ?$

$\vec{P}_1 = -\hat{x}\ 0.40\,kg\ 20.0\,m/s$

$= -\hat{x}\ 8\ kgm/s$

$\vec{p}_2 = \hat{x}\left(30\,m/s \cdot 0.40\,kg\ \underbrace{\frac{\sqrt{2}}{2}}_{\cos 45}\right) + \hat{y}\left(\quad\right)$

$$\boxed{\vec{J} = \vec{p}_2 - \vec{p}_1 = \hat{x}\left(16.5\ kgm/s\right) + \hat{y}\left(8.5\ kgm/s\right)}$$

$\Rightarrow \quad \vec{F} \quad \hat{x}\left(16.5\ kgm/s\right) \quad \hat{y}\left(8.5\ kgm/s\right)$

$$\vec{F} = \frac{\vec{J}}{\Delta t} = \hat{x}\left(\frac{16.5\ \text{kg m/s}}{0.010\ \text{s}}\right) + \hat{y}\left(\frac{8.5\ \text{kg m/s}}{0.010\ \text{s}}\right)$$

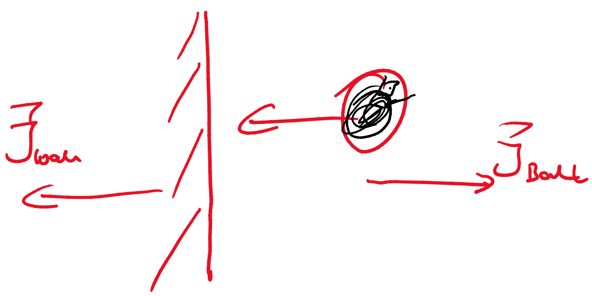
$$= \hat{x}\ 1650\ N + \hat{y}\ 850\ N$$

$$\tan^{-1}\left(\frac{850\ N}{1650\ N}\right) = 27^\circ$$

## Conservation of Momentum

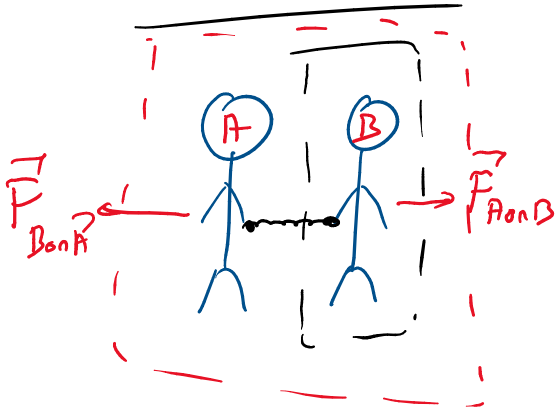
→ Newton's 3rd law → action = reaction

Impulse will be of the same magnitude




→ Internal force: for any system, the forces that the parts of the system exert on each other.

→ External forces: force applied on any part of the system.




$$\Rightarrow \vec{F}_{AB} + \vec{F}_{BA} = \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt}$$

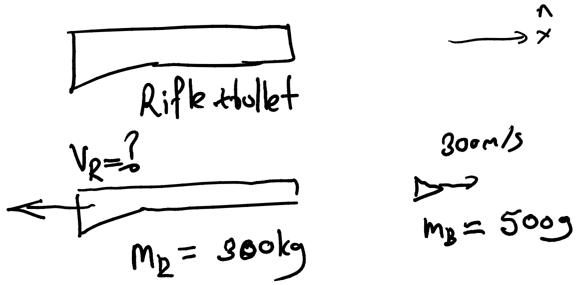
$$= \frac{d(\vec{p}_A + \vec{p}_B)}{dt}$$

$$= 0$$

Ex 8.4)

$P = 0$

Ex 8.4)




$\rightarrow \hat{x}$

$P_x = 0$

$$P_x = 0 = m_B \cdot V_B + (m_R \cdot V_R)$$

$$-150\,kg\,m/s = m_R \cdot V_R$$

$$\boxed{\frac{-150\,kg\,m/s}{m_R} = V_R}$$

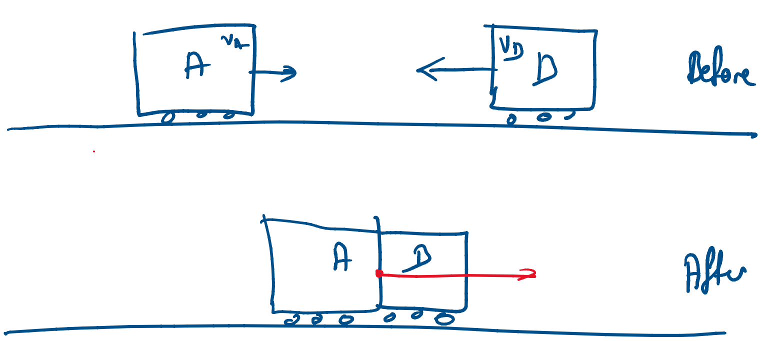
0.5 m/s

## Momentum Conservation & Collisions

Collisions; Elastic $\rightarrow$ conserves energy
Inelastic $\rightarrow$ it does not!

$\Rightarrow$ In any collision in which external forces can be ignored, momentum is conserved and the total momentum before equals to the total momentum after!

## Completely Inelastic Collisions




$$\boxed{m_A \vec{V_A} + m_B \vec{V_B} = (m_A + m_B) \cdot \vec{V_{AB}}}$$

Ex 8.7)

$V_A = 2.0\,m/s$ $\quad V_B = 2.0\,m/s$

Ex 8.9

$V_A = 2.0\,m/s$ $\quad V_D = 2.0\,m/s$

$m_A = 0.50\,kg$ $\quad m_B = 0.30\,kg$

$V_{AB} =$

$1.0\,kg\,m/s\,\hat{y} - 0.60\,kg\,m/s\,\hat{x} = 0.80\,kg \cdot \vec{V}_{AD}$

$$\boxed{\vec{V}_{AD} = 0.50\,m/s\,\hat{x}}$$

$$\Delta K = \underbrace{\frac{1}{2}(m_A + m_B)\,V_{AB}^2}_{0.10\,J} \underbrace{- \frac{1}{2} m_A v_A^2 - \frac{1}{2} m_D v_D^2}_{-1.6\,J}$$

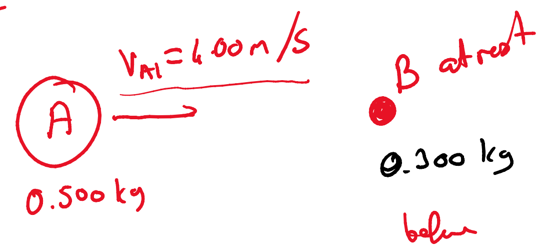
## Elastic Collisions

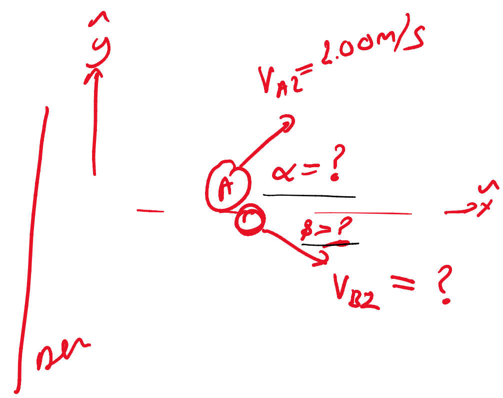
– Kinetic energy is conserved –

> $K_{1i} + K_{2i} = K_{1f} + K_{2f}$

> $\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$

Ex 8.12




Elastic collision

$$\frac{1}{2}\,0.500\,kg\,(4.00\,m/s)^2 + 0 = \frac{1}{2} m_A V_{A2}^2 + \frac{1}{2} m_B V_{B2}^2$$

$$\boxed{V_{B2} = 4.47\,m/s}$$

> $\vec{P}_{A1} = \vec{P}_{A2} + \vec{P}_{B2}$

$4.00\,m/s\;0.500\,kg\,\hat{x} = (m_A V_{A2} \cdot \cos\alpha + m_D V_{D2} \cos\beta)\,\hat{x}$

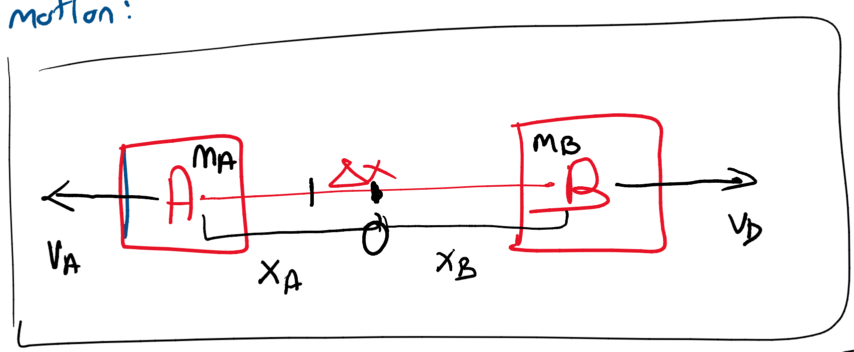
$0\,\hat{y} = (m_A V_{A2} \cdot \sin\alpha - m_D V_{B2} \sin\beta)\,\hat{y}$

$$O_y^n = (m_A v_{A2} \cdot \sin\alpha - m_B v_{B2} \sin 21.5)\,\hat{y}$$

$\alpha = 36.9^\circ \quad \beta = 21.6^\circ$

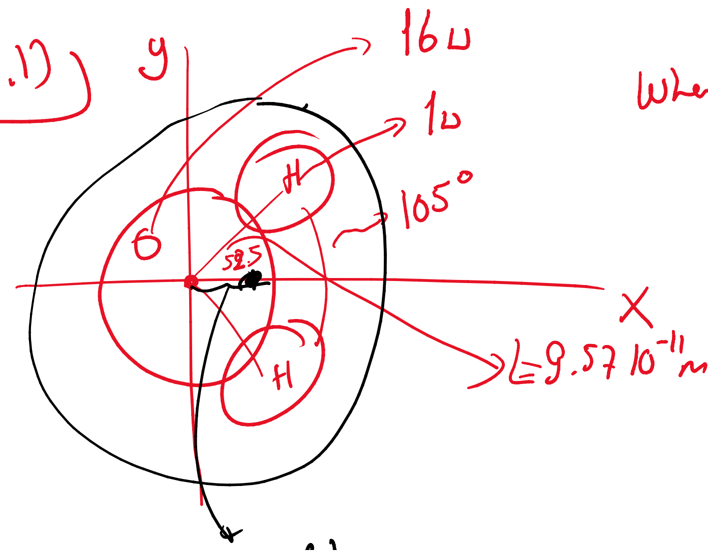
# Center of Mass

↳ When there are multiple parts of a system, looking at the motion of the center of mass makes it easier to study the motion:




$$\underbrace{X_{cm}}_{\text{position of the center of mass}} = \frac{m_A \cdot X_A + m_B \cdot X_B}{(m_A + m_B)} \rightarrow \boxed{\vec{X}_{cm} = \frac{\sum_i m_i \vec{x}_i}{\sum_i m_i}}$$

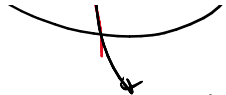
Ex 8.1)




Where is the center of mass:

$$X_{cm} = \frac{16u \cdot 0 + 1u \cdot (\cos 52.5 \cdot L) + 1u(\cos 52.5 \cdot L)}{18u}$$

$$X_{cm} = 0.068L$$



$0.068L$

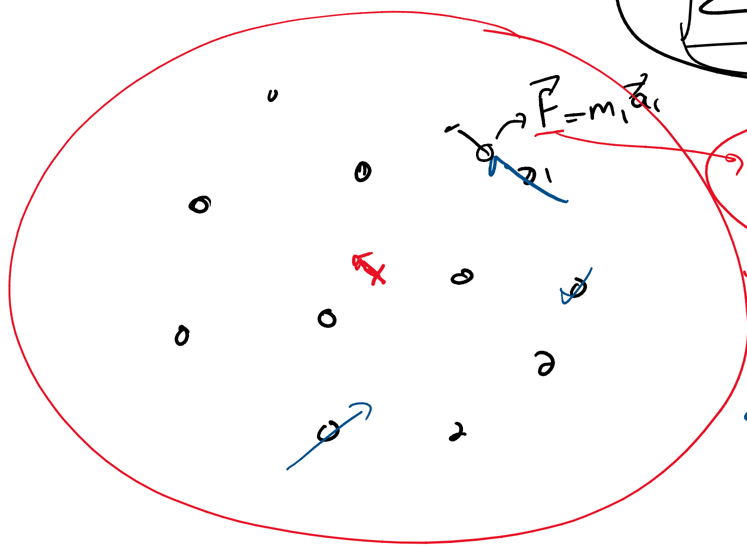
$x_{cm} = 0.068L$

$y_{cm} = 0L$

$\Rightarrow$ Any impact on the parts will effect the centr of mass;

$$\vec{V}_{cm} = \frac{\sum_i m_i \vec{v}_i}{\sum m_i} \Rightarrow \boxed{M\vec{V}_{cm} = \vec{P}}$$

$$\boxed{\sum \vec{F}_{ext} = M\vec{a}_{cm}}$$



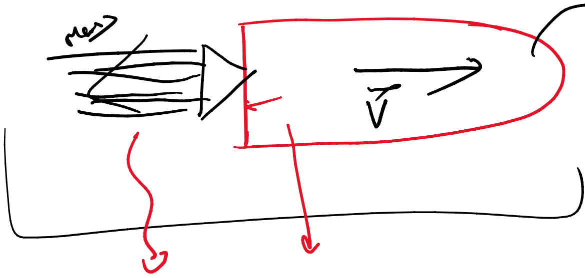

$\vec{F} = M\vec{a}_{cm}$

$\sum \vec{F} = 0$

$\Rightarrow$ When a body or a collection of particles is acted on by external forces, the center of mass moves as though all the mass were concentrated at that point, and it were acted only by a net force equal to the sum of the external forces on the system.

## $\Rightarrow$ Rocket Repulsion

$m(t)$




$m(t)$

$$\frac{d}{dt}\left(\vec{p}\right)$$

$$\left(m v(t)\right)$$

$V - V_{ex} = V_{fuel}$

velocity of exhaust gases

$$m\frac{dv}{dt} = -V_{ex}\frac{dm}{dt}$$

$$\boxed{F = -V_{ex}\frac{dm}{dt}}$$

$$\Downarrow$$

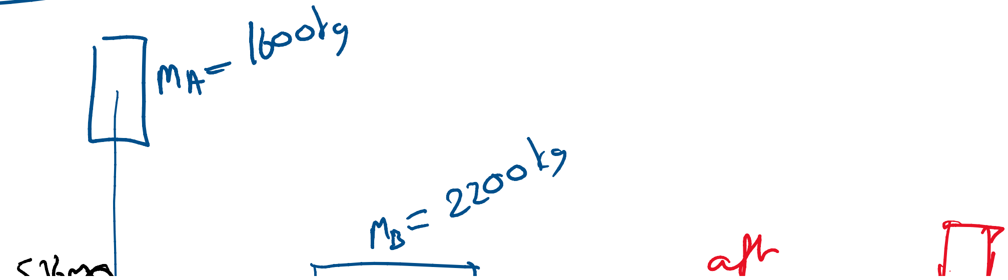
$$ma = -V_{ex}\frac{dm}{dt}$$

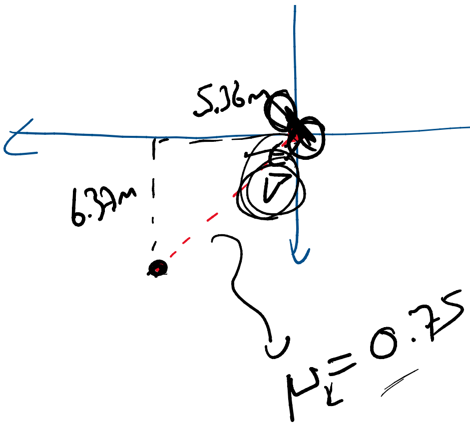
$$\boxed{a = -\frac{V_{ex}}{m}\frac{dm}{dt}}$$

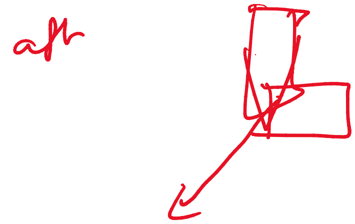
$$\int_{t_0}^{t_f} a\,dt = -V_{ex}\int_{m_0}^{m}\frac{dm'}{m'}$$

$$\boxed{V - V_0 = -V_{ex}\ln\frac{m}{m_0}}$$

## Ex 8.77






How fast was each car travelling just before the collision?

$$K_{AB}^{co} = W_f + \cancel{(1.E\ \text{collision})}$$

$$= ((2200\,kg + 1600\,kg) \cdot 9.8\,m/s^2 \cdot 0.75)$$