Lecture 15

14 Kasım 2019 Perşembe 09:39

Previous lecture:

$$\text{Work} = \underbrace{\Delta K}_{\text{change in kinetic energy}} = \underbrace{-\Delta U}_{} \rightarrow \text{change in potential energy}$$

no dissipative forces (friction, air res) etc.

If a force is conservative:

$$-\vec{\nabla} U = \vec{F}$$

$$\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$\Rightarrow \Delta K + \Delta U = 0$$

> manifestation of conservation of energy

Pr. 7.80

> A proton with mass m moves in on dimension:

$$U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x} \qquad \alpha, \beta > 0$$

a) Plot $U(x)$

b) $v(x)$? position dep. speed. $\rightarrow$ $x_0 = \frac{\alpha}{\beta}$

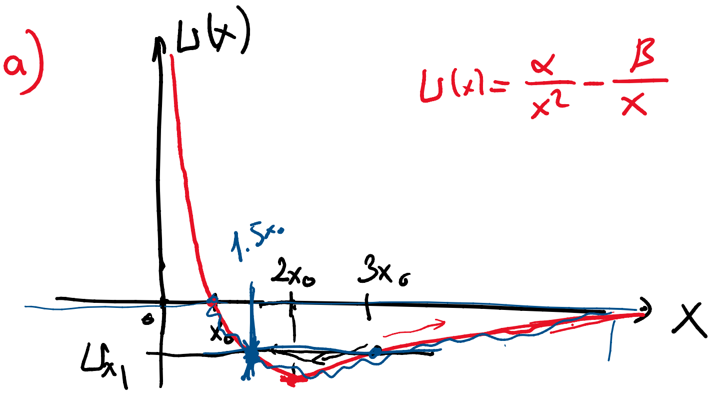
c) ×

d) ×

e) If the proton is released from rest a $x_1 = \frac{3\alpha}{\beta}$, $v(x)$?

f) For each: x_0, x_1 what are the x_{min}, x_{max}?

$U(x)$

a) 

$U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x}$

$U(x) = 0 = \frac{\alpha}{x_0^2} - \frac{\beta}{x_0}$

$x_0 = \alpha/\beta$

$\frac{dU(x)}{dx} = 0 = -2\frac{\alpha}{x^3} + \frac{\beta}{x^2}$

$x_E = \frac{2\alpha}{\beta}$

b) $V(x)$ when we release proton at x_0 from rest.

$-\frac{dU(x)}{dx} = F(x) = ma(x) \longrightarrow V(x)$?

$U(x) < 0$ for $x > x_0$

$U(x) > 0$ for $x < x_0$

$\Delta K = -\Delta U$

$\frac{1}{2}m\left(V(x)^2 - 0^2\right) = -\left(U(x) - 0\right)$

$V(x)^2 = -\frac{2}{m}\left(\frac{\alpha}{x^2} - \frac{\beta}{x}\right) \longrightarrow \boxed{V(x) = \sqrt{\frac{2}{m}\left(\frac{\beta}{x} - \frac{\alpha}{x^2}\right)}}$

~~$V(x) = \sqrt{-\#} = i$~~

e) $x_1 = \frac{3\alpha}{\beta} = 3x_0 \longrightarrow$

$\frac{1}{2}mV(x)^2 = -\left(U(x) - \left(\frac{\alpha}{9x_0^2} - \frac{\beta}{3x_0}\right)\right)$

$V(x) = \sqrt{\frac{2}{m}\left[\left(\frac{\alpha}{9x_0^2} - \frac{\beta}{3x_0}\right) - U(x)\right]}$

$V(x) = \sqrt{\frac{2}{m}\left[\left(\frac{\alpha}{9x_0^2} - \frac{\beta}{3x_0}\right) - \left(\frac{\alpha}{x^2} - \frac{\beta}{x}\right)\right]} \rightsquigarrow 3x_0$

f) X_{min} & X_{max}

f) x_{min} & x_{max}

x_0	$3x_0$
$x_{min} = 0$	$x_{min} = ?$
$x_{max} = +\infty$	$x_{max} = 3x_0$

$$U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x}$$

$$U(3x_0) = \frac{\alpha}{9x_0^2} - \frac{\beta}{3x_0}$$

$$U(3x_0) = U(x)$$

$$\left(\frac{\alpha}{9x_0^2} - \frac{\beta}{3x_0}\right) = \frac{\alpha}{x^2} - \frac{\beta}{x}$$

$$x_0 = \alpha/\beta$$

$$1x^2 - \frac{9\alpha}{2\beta}x + \frac{9\alpha^2}{2\beta^2} = 0$$

$$x_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{\pm} = \frac{9\alpha}{4\beta} \pm \frac{3\alpha}{4\beta}$$

$$x_+ = \frac{12\alpha}{4\beta} = 3x_0$$

$$x_- = \frac{6\alpha}{4\beta} = 1.5x_0$$