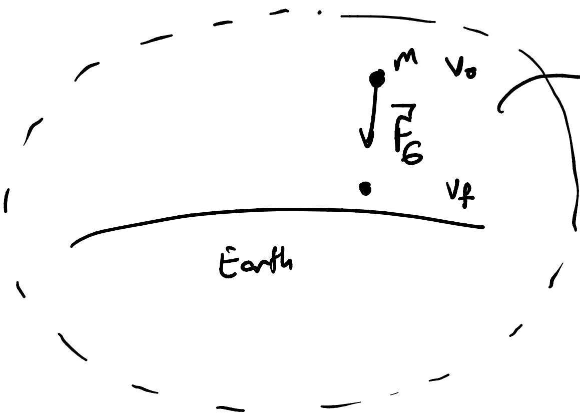
Lecture 14

11 Kasım 2019 Pazartesi 10:41

Chapter 7

Potential Energy and Energy Conservation



Gravity does work on the object

$$\underbrace{W = \Delta K = \int_{x_1}^{x_2} \vec{F} \cdot d\vec{s}}$$

One alternative way to think:

"potential energy" ⇄ kinetic energy

↓

Total mechanical Energy

$$\boxed{\text{Total Energy} = \text{Kinetic Energy} + \text{Potential Energy}}$$

$$W = \vec{F} \cdot \vec{s} = m\vec{g} \cdot \vec{s} = mg(y_0 - y_f) \qquad \boxed{U_{grav} = mgy}$$

$$W_{grav} = U_{g0} - U_{gf}$$

$U_{g0} = mgy_0$

$U_{gf} = mgy_f$

$$\boxed{W_G = -\Delta U}$$

$\Delta U = U_{gf} - U_{g0}$

⇓

$$\boxed{\Delta K = W_G = -\Delta U}$$

* This only true if the gravity is the sole source of force!

⇓

$$\boxed{K_1 + U_1 = K_2 + U_2}$$

⇒ What if there are other forces;

$$W_{other} + W_G = \Delta K$$

→ What if ...

$$W_{other} + \underbrace{W_G}_{-\Delta U} = \Delta K$$

$$\boxed{W_{other} + K_1 + U_1 = K_2 + U_2}$$

Example 7.2)

Suppose you throw a ball upwards, your hand moves 0.5m upwards while you throw the ball with a constant force. When it leaves your hand it has a speed of 20.0 m/s. Ball weighs 0.145 kg

a) Find the force your hand exerts on the ball assuming a constant force:

① 

$$\vec{F} - \vec{w} = m\vec{a}$$

$$\overset{20.0\,m/s}{V_f^2} = \overset{0}{V_0^2} + 2a\overset{0.5m}{x}$$

$$a =$$

$$F = m(a+g)$$

$$\boxed{F = 59\,N}$$

② $K_2 = \frac{1}{2}mv^2 = 29.0\,J$

$U_2 = mg\underbrace{h}_{0.5m} = 0.71\,J$

$K_1 = 0\,J$

$\boxed{U_1 = 0\,J} \rightarrow \Delta U$

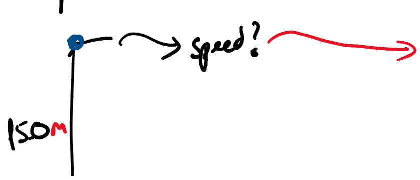
$$W_{other} + \overbrace{K_1 + U_1}^{0\,J} = K_2 + U_2$$

$$W_{other} = 29.7\,J$$

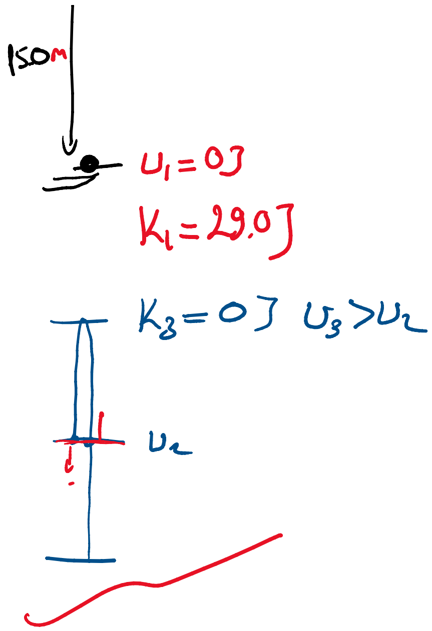
$$\vec{F}\cdot\underset{0.5m}{\vec{s}} = 29.7\,J$$

$$\boxed{F = 59\,N}$$

b) Find the speed of the ball after 15.0 meters above the point where it leaves the hand:



$$K_1 + U_1 = K_2 + U_2$$



$$K_1 + U_1 = K_2 + U_2$$

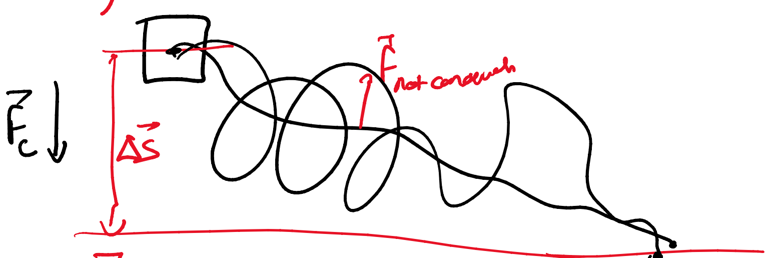
$$29.0J = \frac{1}{2}(0.145kg)\cdot v_2^2 + \underbrace{mg\cdot(15.0m)}_{21.3J}$$

$$7.7J = \frac{1}{2}\,0.145kg\cdot v_2^2$$

$$\left(v_2\right)^2 = \left(\pm\sqrt{\frac{2\times 7.7J}{0.145kg}}\right)^2$$

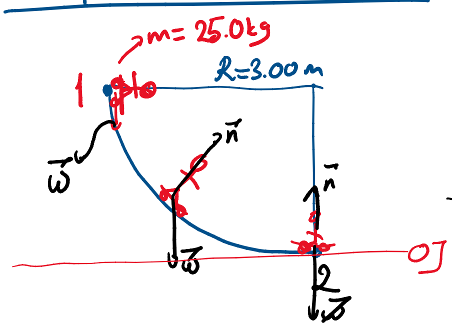
$$\boxed{v_2 = \pm 10\,m/s}$$

$\Rightarrow$ Any conservative force:



$$W = \vec{F}_c \cdot \vec{\Delta s}$$

Example 7.4 & 7.5



a) Speed at the bottom
b) Normal force at the bottom
} assume no friction

c) If his speed is 6.00 m/s, what is the energy lost to dissipative forces?

a)

$K_1 = 0J$

$U_1 = mgh$ (h = 3.00m)

$K_2 = \frac{1}{2}(25.0kg)v_2^2$

$U_2 = 0J$

$$K_1 + U_1 = K_2 + U_2$$

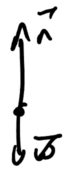
$$\cancel{m}gh = \frac{1}{2}\cancel{m}v_2^2$$

$$v_2 = \sqrt{2gh}$$

$$\boxed{v_2 = 7.67\,m/s}$$

$\boxed{v_2 = 7.67 \text{ m/s}}$

b) Normal force at point 2?



$$\vec{n} - \vec{w} = m\frac{v^2}{R}(\hat{n})$$

$$n - mg = m\frac{2g\cancel{h}}{\cancel{R}} \quad h = R$$

$$\boxed{n = 3mg}$$

c) What if $v_2 = 6.00$ m/s

$$\underbrace{K_1 + U_1}_{785\text{ J}} + W_{other} = \underbrace{K_2 + U_2}_{450\text{ J}}$$

$K_1 = 0$ J, $U_1 = mgh$, $K_2 = \frac{1}{2}(25.0\text{ kg}) \times (6.00\text{ m/s})^2$

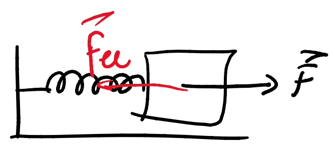
$$785\text{ J} + W_{other} = 450\text{ J}$$

$$\boxed{W_{other} = -285\text{ J}}$$

energy lost to friction

⇒ Elastic Potential Energy

"Total energy is conserved"



Work done on a spring $\quad W = \int_{x_1}^{x_2} kx\,dx = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$

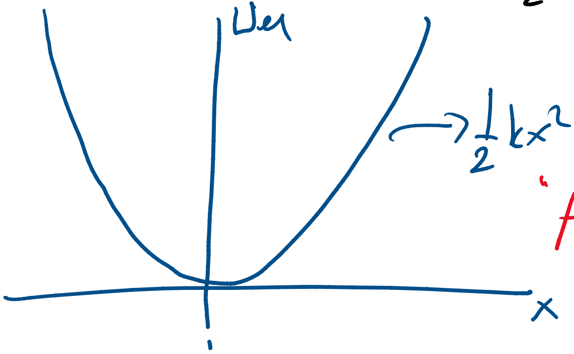
$\frac{1}{2}kx^2$

Work done by a spring $\quad \boxed{W_{el} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2}$

$$\boxed{W_{el} = -\Delta U} = -(U_2 - U_1)$$

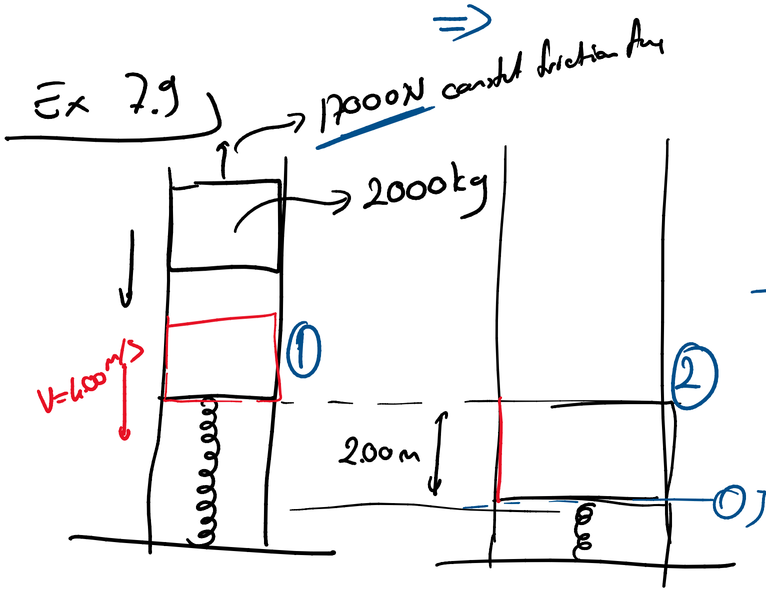
[illegible] $\frac{1}{2}kx^2$

$U_2 = \frac{1}{2} k x_2^2 \; , \; U_1 = \frac{1}{2} k x_1^2 \; > \; \boxed{U_{el} = \frac{1}{2} k x^2}$



"Harmonic Oscillator"

Ex 7.9



What k should we have to stop the elevator in 2 meters?

$> \; K_1 = \frac{1}{2}(2000\,kg)\cdot(4.00\,m/s)^2$

$U_{1g} = 19600\,N \cdot 2.00\,m$

$U_{1s} = 0\,J$

$> \; K_2 = 0\,J \quad U_{1s} = \frac{1}{2} k (2.00)^2$

$U_{1g} = 0\,J$

$$\underbrace{K_1 + U_{1g} + U_{1s}}_{} + W_{other} = K_2 + U_{2g} + U_{2s}$$

16000 J 39200 J 0 J $(-17000\,N \cdot 2.00\,m)$

$$\left(\frac{21200\,J \times 2}{4.00\,m^2}\right) = k \qquad = \frac{1}{2} k \, 4.00\,m^2$$

$$= 10600\,N/m$$

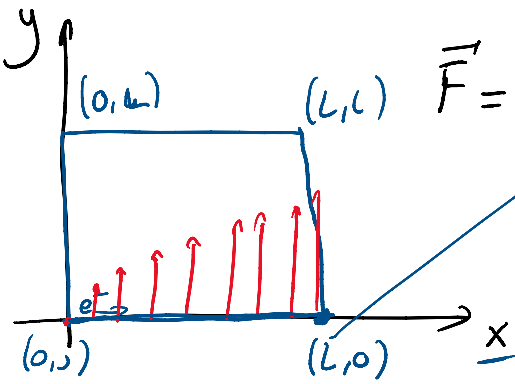
Conservative vs Non-Conservative

Conservative vs Non-Conservative forces

Conservative	Non-Conservative
∘ $E = K + U$	→ Dissipates energy regardless of the direction of the motion.
∘ Reversible	→ No potential energy
∘ Independent of the path it takes.	→ Only the total energy is conserved.

$E = mc^2$ "Entropy"

Ex |



$\vec{F} = \underbrace{C}_{\text{constant}} x\, \hat{j}$ → Is conservative or not?

$$W = \oint \vec{F}\cdot d\vec{l} = \underbrace{\int_{(0,0)}^{(L,0)} \vec{F}\cdot d\vec{l}}_{0J} + \underbrace{\int_{(L,0)}^{(L,L)} \vec{F}\cdot d\vec{l}}_{Cx \cdot L \;\to\; CL^2} + \underbrace{\int_{(L,L)}^{(0,L)} \vec{F}\cdot d\vec{l}}_{0J} + \underbrace{\int_{(0,L)}^{(0,0)} \vec{F}\cdot d\vec{l}}_{0J} = W$$

$$\boxed{W = CL^2}$$

not a conservative force!

⇒ Force & Potential Energy:

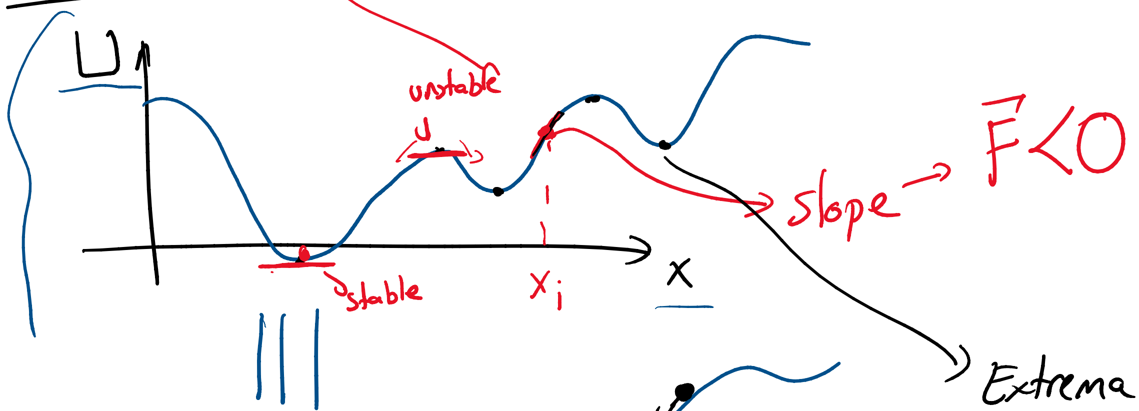
nabla

$$W = -\Delta U \qquad \overbrace{\int_1^2 \vec{F}\cdot d\vec{r}}^{W} = -\Delta U \quad \longrightarrow \quad \boxed{-\vec{\nabla} U = \vec{F}}$$

$$\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)U$$

$$F\,dx = -dU$$

$$\boxed{F = -\frac{dU}{dx}} \qquad -\left(\hat{x}\,\frac{\partial U(x,y,z)}{\partial x} + \hat{y}\,\frac{\partial U(x,y,z)}{\partial y} + \hat{z}\,\frac{\partial U(x,y,z)}{\partial z}\right) = \vec{F}$$



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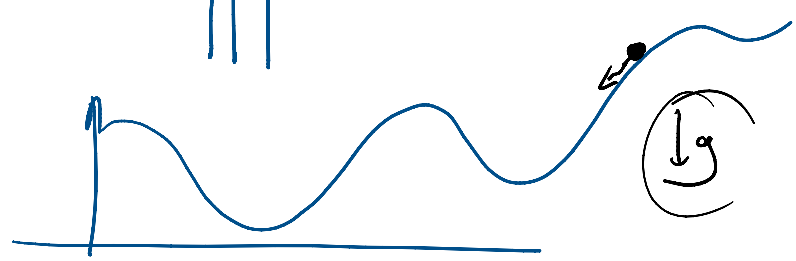

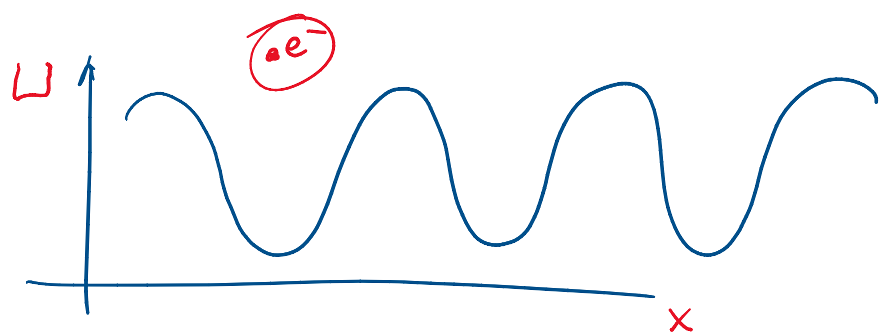

$$\Rightarrow \quad U = \frac{1}{2}kx^2 \qquad \text{Harmonic oscillator}$$

$$F = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx \qquad \text{"Hooke's law"}$$

$$F = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$ Hooke's law

2D - Harmonic Oscillator

$$U(x,y) = \frac{1}{2}k(x^2+y^2)$$

$$\vec{F} = ? \quad \vec{F} = -\left(\hat{x}\frac{\partial U}{\partial x} + \hat{y}\frac{\partial U}{\partial y}\right)$$

$$= -\hat{x}kx - \hat{y}ky$$