Lecture5

7 Ekim 2019 Pazartesi 10:38

> Motion in two or three dimensions

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

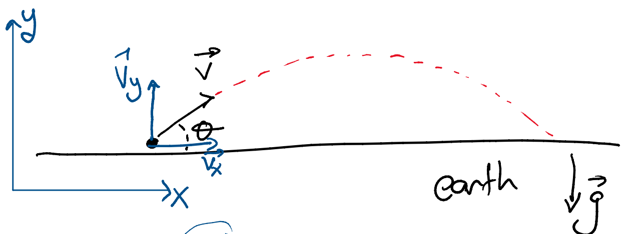
$$\vec{V} = \frac{d\vec{r}}{dt} = \dot{\vec{r}} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

⇒ Projectile Motion

> a very specific motion under the influence of gravity.

18th, 19th century



$$\vec{a} = 0\hat{i} + (-g)\hat{j}$$

gravitation acceleration

assuming no air resistance

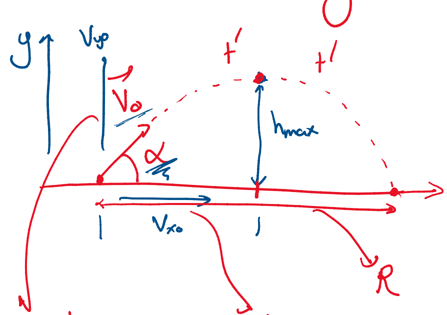
$$\vec{V} = V_x\hat{i} + V_y\hat{j} \rightarrow \vec{V}_0 = V_{x0}\hat{i} + V_{y0}\hat{j}$$

time = 0

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\boxed{\begin{aligned} x &= x_0 + V_{x0}\cdot t \\ y &= y_0 + V_{y0}t - \tfrac{1}{2}gt^2 \end{aligned}}$$

Ex) Maximum range?



$|\vec{V}_0|.\sin\alpha$ $|\vec{V}_0|\cos\alpha$

$v_y(t') = 0$ $|\vec{V}_0|$

$$v_y(t') = V_{y0} - gt' = 0$$

$$\boxed{t' = \frac{V_{y0}}{g}} = \frac{|\vec{V}_0|.\sin\alpha}{g}$$

$$h = 0 + |\vec{V}_0|\sin\alpha\, t' - \frac{1}{2}gt'^2$$

$|\vec{V_0}| . \sin\alpha$ $\qquad |\vec{V_0}| \cos\alpha$ $\qquad R$

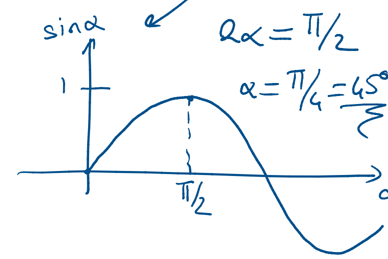
$$h_{max} = 0 + |\vec{V_0}| \sin\alpha \, t' - \frac{1}{2} g t'^2$$

$$= \frac{|\vec{V_0}|^2 \sin^2\alpha}{g} - \frac{1}{2} g \cdot \frac{|\vec{V_0}|^2 \sin^2\alpha}{g^2}$$

$$\boxed{h_{max} = \frac{1}{2} \frac{|\vec{V_0}|^2 \sin^2\alpha}{g}}$$

$$R = \underbrace{V_{x0}}_{} \cdot (2t')$$

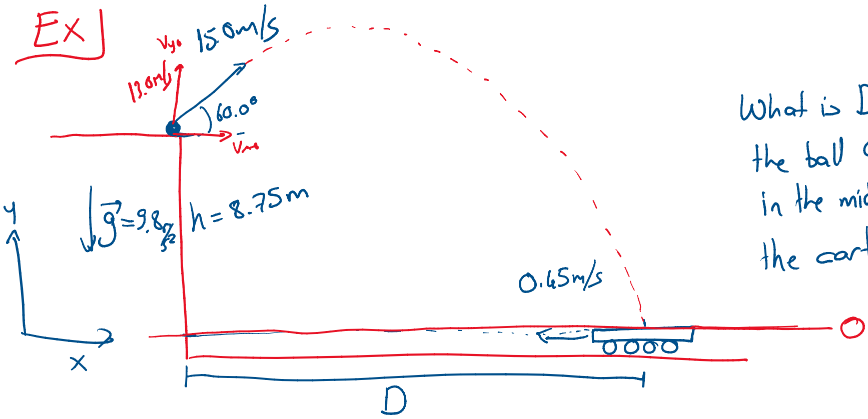
$$= |\vec{V_0}| \cos\alpha \cdot \frac{2|\vec{V_0}| \sin\alpha}{g} = \frac{|V_0|^2}{g} \underbrace{2\sin\alpha\cos\alpha}_{\sin 2\alpha} = \boxed{\frac{|V_0|^2}{g} \sin 2\alpha = \text{Range}}$$



$2\alpha = \pi/2$

$\alpha = \pi/4 = 45°$

Ex



What is D so that the ball can land in the middle of the cart?

$$v(t) = V_{y0} - g t$$

integral

$$y(t) = y_0 + V_{y0} t - \frac{1}{2} g t^2$$

$$= \underline{8.75\,m + 13.0\,m/s \cdot t - \frac{1}{2} 9.8 \frac{m}{s^2} t^2}$$

$$V_{y0} = 15.0\,m/s \cdot \sin 60.0°$$
$$= 13.0\,m/s$$

$$V_{x0} = 15.0\,m/s \cdot \cos 60.0°$$
$$= 7.5\,m/s$$

$$y(t) = 0 \Rightarrow \quad t^2 - \frac{13.0\,s}{4.9} \cdot t - \frac{8.75}{4.9} s^2 = 0$$

$$t_\pm = \frac{-b}{2a} \pm \frac{1}{2a}\sqrt{b^2 - 4ac}$$

$$\boxed{ax^2 + bx + c = 0}$$

$a = 1$, $b = -\frac{13.0}{4.9} s$, $c = -\frac{8.75}{4.9} s^2$

$$t_\pm = 1.3\,s \pm 1.5\,s$$

$$\boxed{t = 2.8\,s}$$

$$x(t) \rightarrow x(2.8\,s) = 7.5\,m/s \cdot 2.8\,s = 21.0\,m$$

D

$$21.0\,m + V_c \cdot 2.8\,s = 22.3\,m$$

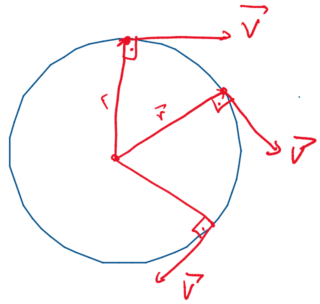
$V_c = 0.45\,m/s$

D

$$21.0\,m + \overset{0.45\,m/s}{v_c} \cdot 2.8\,s = 22.3\,m$$

Motion in a circle

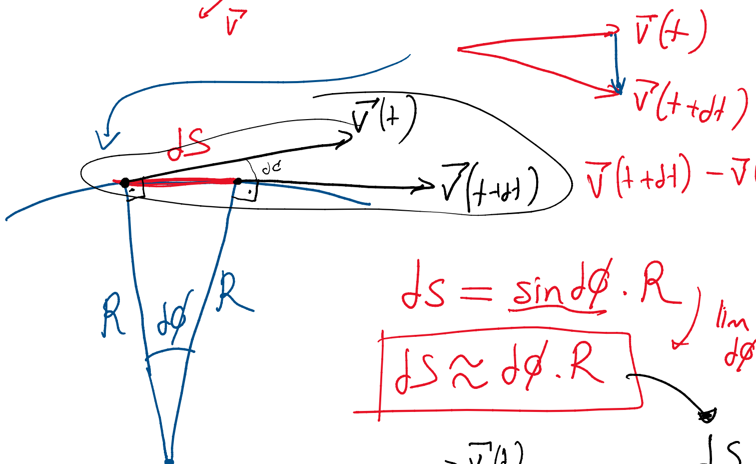
Uniform circular motion



$$|\vec{v}(t)| = \text{constant}$$

$$\frac{d\vec{v}(t)}{dt} = ?$$

$$\frac{d(f(t))}{dt} = 0 \qquad C$$

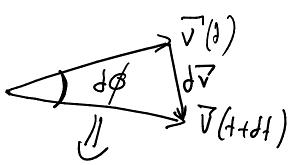


$$\vec{v}(t+dt) - \vec{v}(t)$$

$$ds = \sin d\phi \cdot R$$

$\lim_{d\phi \to 0}$

$$\boxed{ds \approx d\phi \cdot R}$$



$$|\vec{v}|\, d\phi = |d\vec{v}|$$

$$|\vec{v}| \cdot \frac{d\phi}{dt} = \frac{|d\vec{v}|}{dt}$$

$$|\vec{v}| \cdot \frac{d\phi}{dt} = |\vec{a}|$$

$$\omega = \frac{|\vec{v}|}{R}$$

$$\frac{ds}{d\phi} = R$$

$$\frac{ds}{dt} \cdot \frac{dt}{d\phi} = R$$

$$\boxed{\frac{d\phi}{dt} = \frac{1}{R}\frac{ds}{dt}}$$

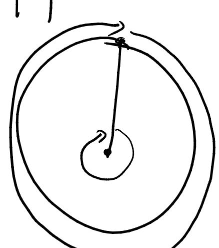
$$\Rightarrow \frac{d\phi}{dt} = \omega$$

angular velocity

$$\boxed{\omega \cdot R = \frac{ds}{dt}} = |\vec{v}|$$

$$\boxed{\omega \cdot R = |\vec{v}|}$$

$$\boxed{2\pi = \omega \cdot T}$$

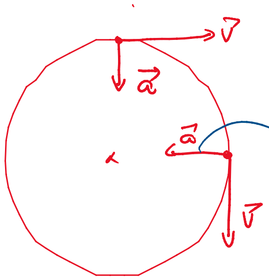


$$|2\pi = \omega \cdot T|$$

T → period of a revolution

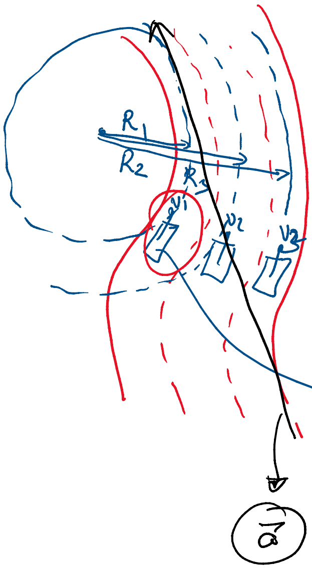
$$\Downarrow$$

$$\boxed{v = \frac{2\pi R}{T}}$$



→ Centripetel acceleration

⇒ Ex, Dangerous driving



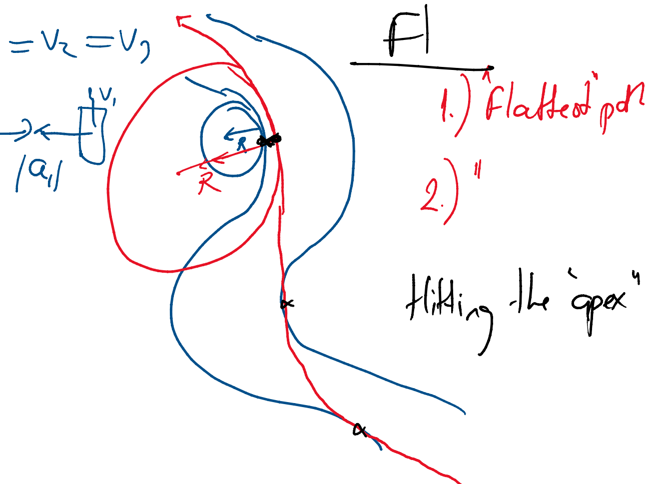
Centripetal acceleration?

$$R_3 > R_2 > R_1$$

$$|\vec a| = ? = \frac{|\vec v|^2}{R}$$

$$|\vec a_1| > |\vec a_2| > |\vec a_3|$$

$$v_1 = v_2 = v_3$$



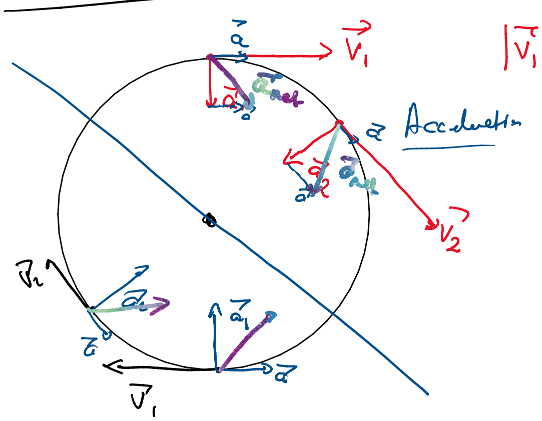
F1

1.) "Flattest" path

2.) "

Hitting the "apex"

Non-uniform Circular Motion



$|\vec v_1| \neq$ const in time

Acceleration

$|\vec v_2| > |\vec v_1|$

Deceleration

$$\underbrace{\frac{d|\vec v|}{dt}}_{\text{tangential acceleration}} \neq \underbrace{\left|\frac{d\vec v}{dt}\right|}_{|\vec a|}$$

$$|\vec a| \rightarrow \sqrt{a_{tan.}^2 + a_{centripetal}^2}$$

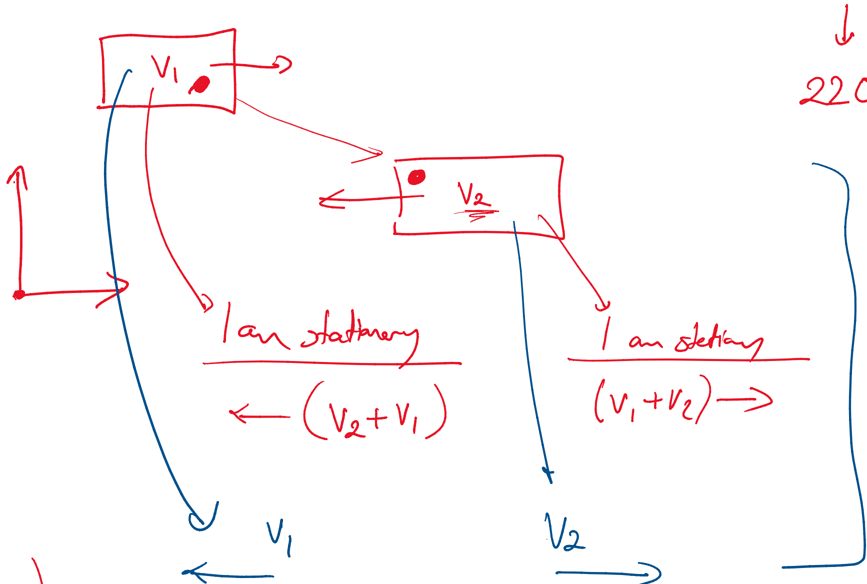
acceleration $\sqrt{a_{tan.}^2 + a_{centripetal}^2}$

Relative Motion

> What is our reference frame? → 80 km/s (1/10,000 km/h)

↓

220 km/s



→ Newton

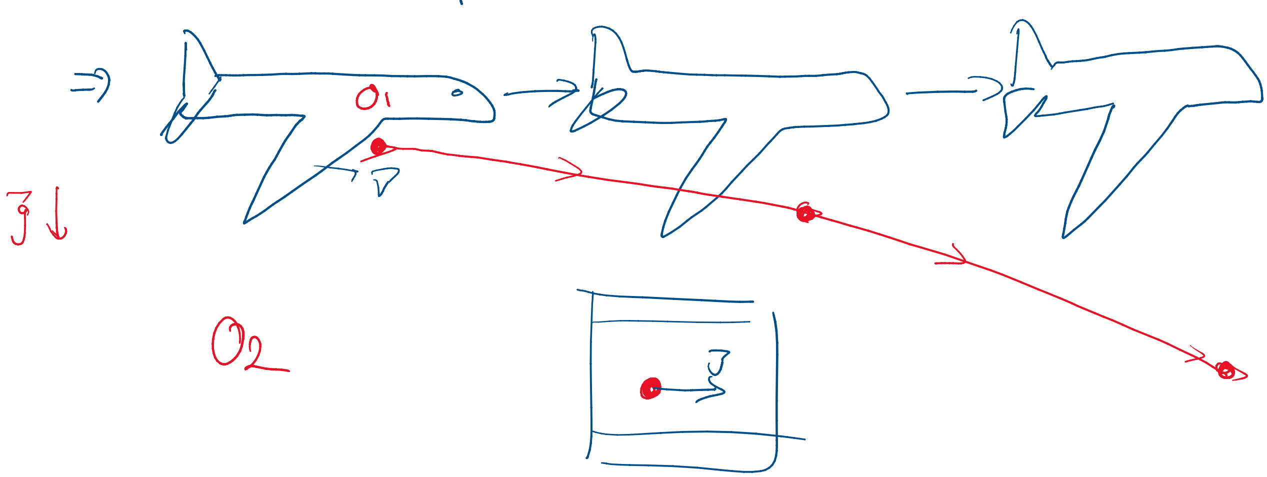
↓

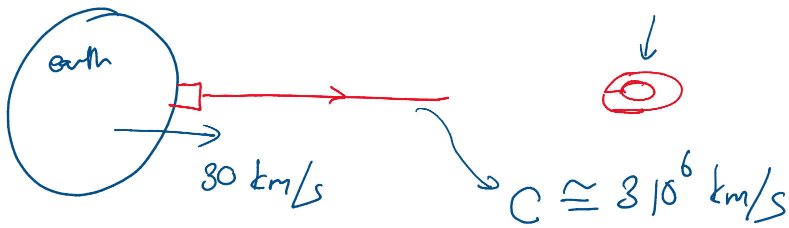
Newton's bucket ? problem

This is Gallilean Relativity

↳ The concept of relativity has a central role in physics:

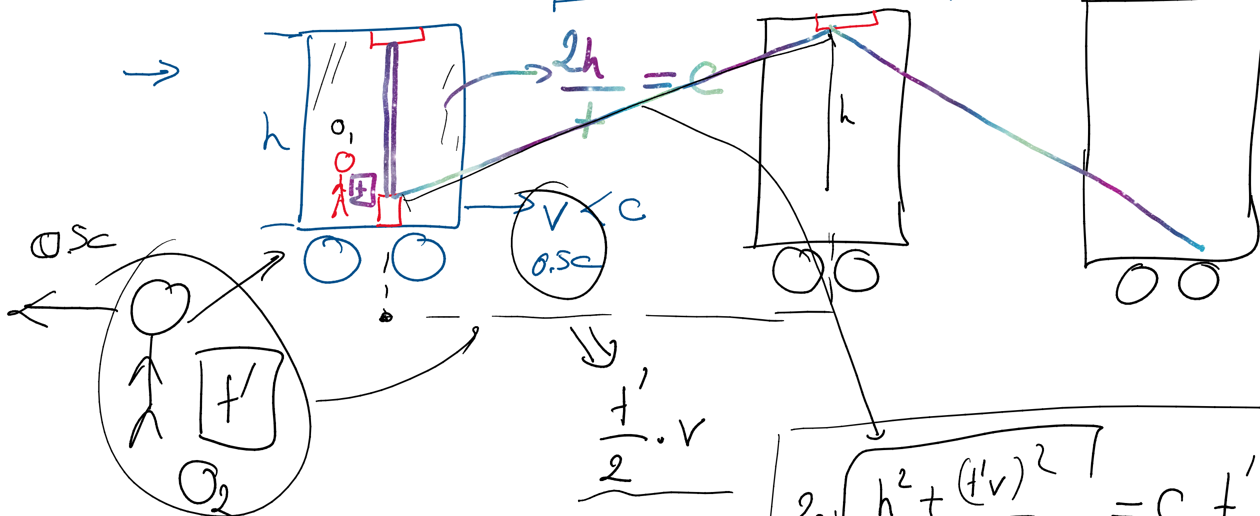
↳ Are we the center of the universe?





$\boxed{c + 30\,km/s}$

Gedanken Exp.



$$\boxed{2 \cdot \sqrt{h^2 + \frac{(t'v)^2}{4}} = c \cdot t'}$$

$$O_1 \Rightarrow \quad c = \frac{2h}{t}$$

$$O_2 \Rightarrow \quad c = \frac{2 \cdot \sqrt{h^2 + \frac{(t'v)^2}{4}}}{t'}$$

L

1898

Einstein

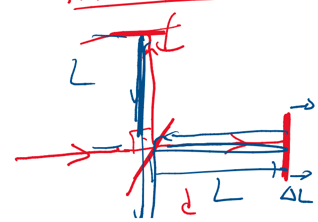
LIGO

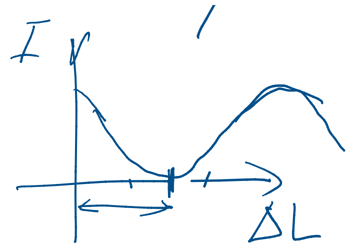
$\boxed{10^{-23}\ m}$

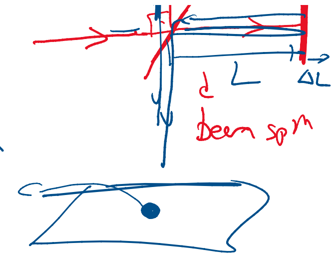
Michelson

Morley experim

Interferometer







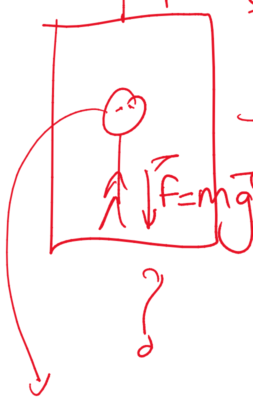
Gravitation

$$\boxed{t = \frac{t'}{\sqrt{1 - v^2/c^2}}}$$

1905

1915 General relativity

$|\vec{a}| = |\vec{g}|$



Gravity

Accelerating object

Space time

$M \rightarrow \leftarrow m$