Lecture 3

30 Eylül 2019 Pazartesi 10:31

> Vectors & Vector Algebra

↳ Addition & Subtraction

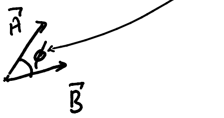
→ Vector Products

- Scalar Product (dot)

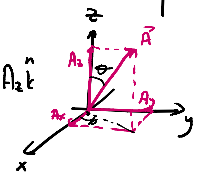
↓

$\vec{A}\cdot\vec{B} = C$

$\vec{A}\cdot\vec{B} = AB\cdot\cos\phi \Rightarrow \hat{i}\cdot\hat{i} = 1$, $\hat{i}\cdot\hat{j} = 0$



$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$



$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

$\vec{A}\cdot\vec{B} = AB\cos\phi$

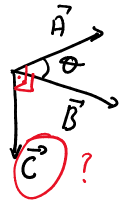
$$\cos\phi = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

$A = |\vec{A}|$ $B = |\vec{B}|$

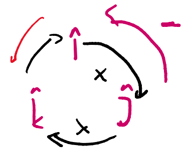
- Vector Product (cross) $\Longleftrightarrow$ Right hand rule

↓

$\vec{A}\times\vec{B} = \vec{C} \rightarrow C = AB\sin\theta$



$\vec{C}$?



$\vec{A}\times\vec{B} = -(\vec{B}\times\vec{A})$

non-commutative operation

$\vec{C} = \vec{A}\times\vec{B}$

$= (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= A_xB_x(\hat{i}\times\hat{i}) + A_xB_y\underbrace{(\hat{i}\times\hat{j})}_{\hat{k}} + A_xB_z\underbrace{(\hat{i}\times\hat{k})}_{-\hat{j}}$

$+ A_yB_x\underbrace{(\hat{j}\times\hat{i})}_{-\hat{k}} + A_yB_z\underbrace{(\hat{j}\times\hat{k})}_{\hat{i}}$

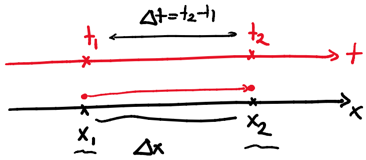
$+ A_zB_x\underbrace{(\hat{k}\times\hat{i})}_{\hat{j}} + A_zB_y\underbrace{(\hat{k}\times\hat{j})}_{-\hat{i}}$

$\vec{C} = \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$

Chapte 2 - Motion along a straight line

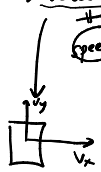
- Displacement
- Time

⇒ velocity ≠ speed

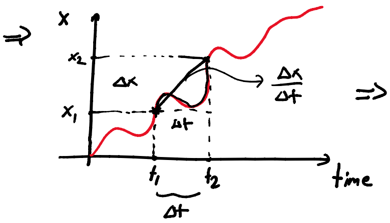


$\Delta t = t_2 - t_1$

$\Delta x = x_2 - x_1$
$\Delta t = t_2 - t_1$ → average velocity $\frac{\Delta x}{\Delta t} = V_{av}$



⇒



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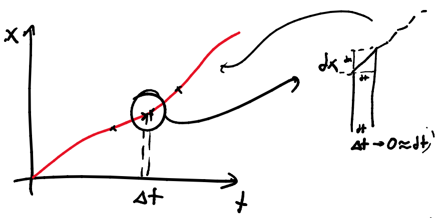


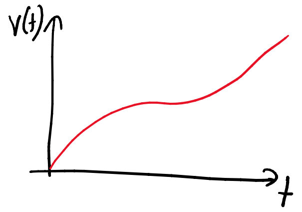
limit $\lim_{\Delta t\to 0} \frac{\Delta x}{\Delta t} = \underbrace{\frac{dx}{dt}}_{\text{derivative}}$

⇒ Instantaneous velocity

$v(t) = \frac{d}{dt}x(t) \ldots v(t)$

$\Rightarrow$ Instantaneous velocity

$$x(t) \rightsquigarrow \underline{\frac{d}{dt}x(t) = v(t)}$$



$$\dot{x}(t) = v(t)$$

$\Rightarrow$

$$\frac{d}{dt}\,v(t) = a(t) \quad \text{acceleration}$$

$$\frac{d}{dt}\,\frac{d}{dt}\,x(t) = \underline{\frac{d^2 x(t)}{dt^2}} = a(t) = \dot{v}(t)$$

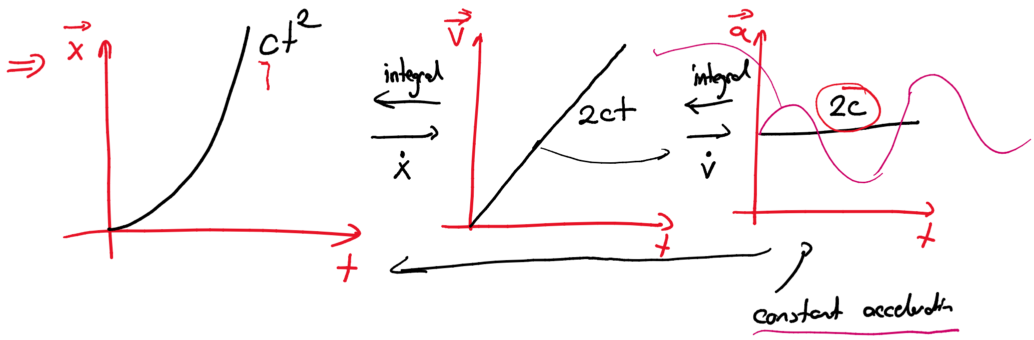
$$\ddot{x}(t) = a(t)$$

Jerk: $\dfrac{d^3 x(t)}{dt^3}$

never used in physics!

acc.

$\Rightarrow$ rate of change of velocity in time

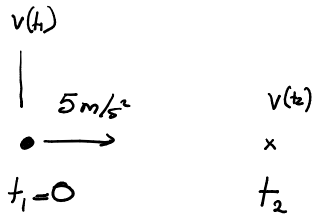


$$\Rightarrow \quad a(t) = \frac{d\,v(t)}{dt} \quad \Rightarrow \quad a(t)\,dt = dv$$

$$\underbrace{\int_{t_1}^{t} a(t)\,dt}_{\int_{t_1}^{t} a\,dt} = \int_{v_1}^{v(t)} dv$$

constant acceleration

$$\int_{t_1}^{t} a\,dt = v(t) - v_1$$



$$a.t - a.t_1 = v(t) - v_1$$

$$\boxed{v(t) = a.t + v_1 - at_1}$$

$$\Downarrow$$

$$\boxed{v(t) = a.t + v(0)}$$

velocity at time=0

$$\frac{dx}{dt} = a.t + v(0) \quad \rightsquigarrow \quad \int_{x_0}^{x} dx = \int_{0}^{t'} \big(at + v(0)\big)\,dt$$

$$x - \overset{x(0)}{x_0} = \frac{1}{2}at'^2 + \underset{v_0}{v(0)}\,t'$$

How position of

$$x - x_0 = \frac{1}{2}at^2 + \underbrace{v(0)}_{v_0}t$$

$$\Rightarrow \boxed{x(t) = x_0 + v_0 t + \frac{1}{2}at^2}$$

→ How position of an object changes in time when under constant acceleration!

$x_0 = 0$

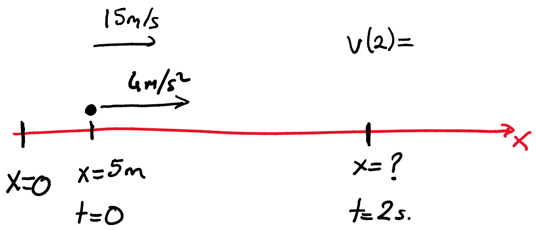
$v_0 = 0$

$$x(t) = \underbrace{\frac{1}{2}a}_{c}\,t^2$$

$\frac{1}{2}a = c$

$a = 2c$

⇒



$v(2) =$

$$x(t) = 5m + 15\,m/s \cdot 2s + \frac{1}{2}\,4\,m/s^2\,(2s)^2$$

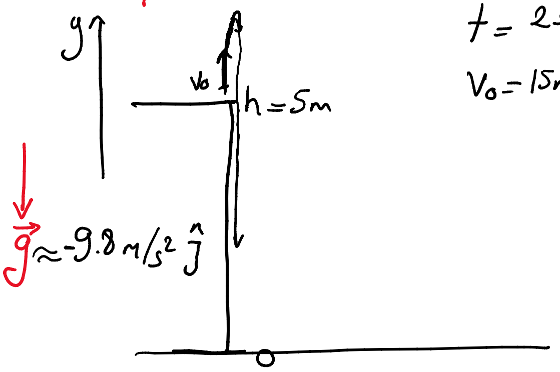
$$= 5m + 30m + 8m$$

$$= 43m$$

$$v(t) = 15\,m/s + 4\,m/s^2 \cdot 2s.$$

$$= 23\,m/s$$

⇒ Free-fall problems



$t = 2s.$

$V_0 = 15m/s$

$$y(t) = 5m + 15\,m/s \cdot 2s + \frac{1}{2}\left(-9.8\,m/s^2\right)(2s)^2$$